Studying the internal structure and dynamics of the Sun

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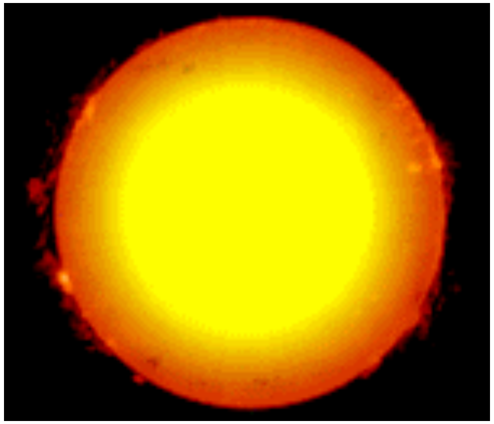
HELIOSEISMOLOGY

Helios: Classical Greek for the Sun

Seismos: Classical Greek for tremors

Logos: Classical Greek for reasoning or discourse



The Sun and the stars oscillates in *normal* modes.

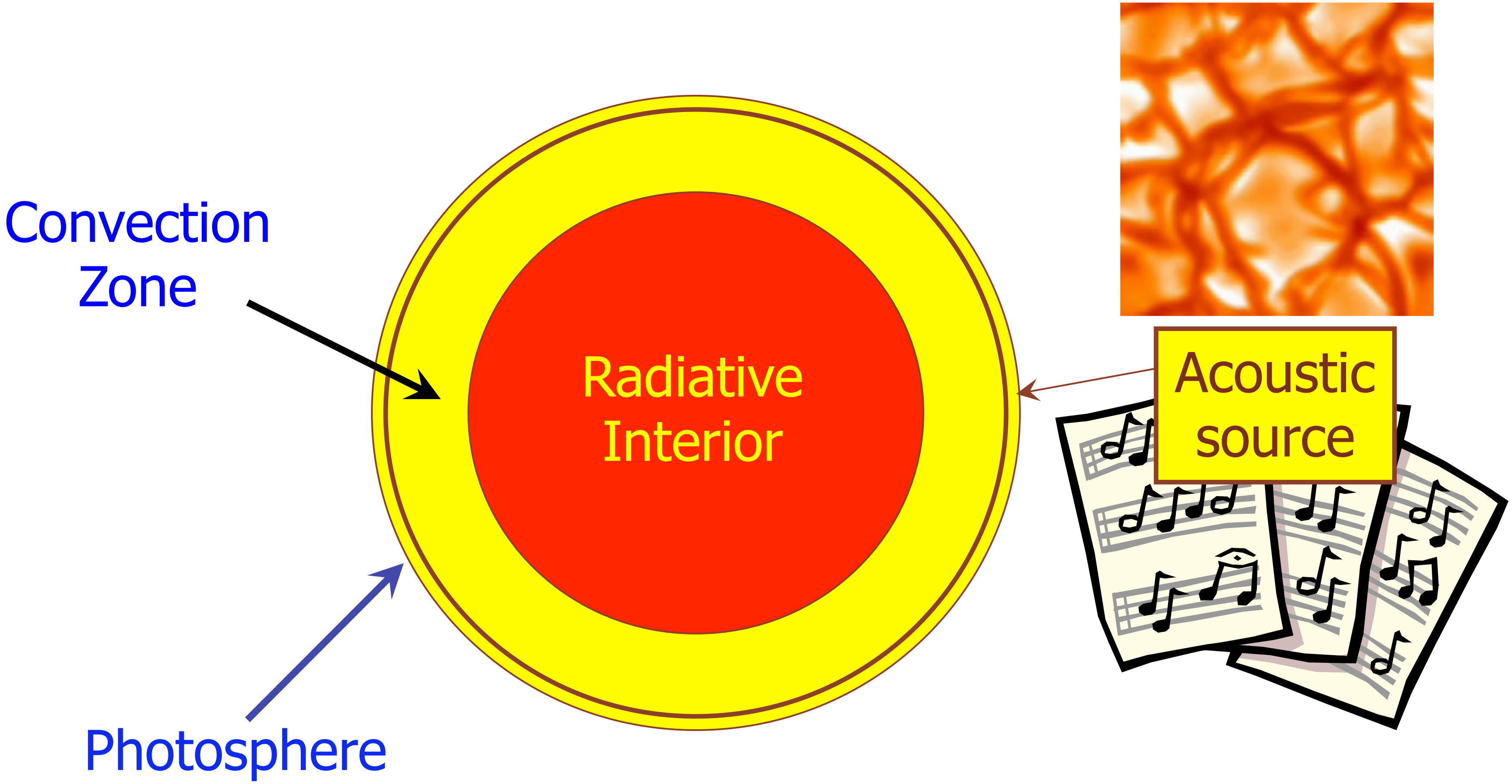
Normal modes in an oscillating system are special solutions where all the parts of the system are oscillating with the same frequency (called "normal frequencies" or "allowed frequencies").

Stellar oscillations are *standing waves*.

A standing wave, also known as a stationary wave, is a wave that remains in a constant position.

This phenomenon can occur because the medium is moving in the opposite direction to the wave, or it can arise in a stationary medium as a result of interference between two waves travelling in opposite directions.

In the Sun, sound waves are generated at top of Convection Zone...



A Crash Course in Helioseismology

- **The Sun oscillates in millions of different modes.**
- **The oscillations are linear and adiabatic.**
- **All observed modes are acoustic i.e., *p*-modes or surface modes i.e. *f*-modes.**
- **Each mode is characterized by three numbers:**

(1) *n*: the radial order, the number of nodes in the radial direction

(2) *l*: the degree, the number of nodal circles on the circumference

(3) *m*: the azimuthal order. m goes from +*l* to -*l*

|*m*|= no. of node circles crossing a latitude

***l*-|*m*|=no. of node circles crossing a longitude.**

l=2, m=1

l=3 ,m=0

l=3, m=−3

- **If the Sun were spherically symmetric and did not rotate, all modes with the same *l* and *n* but different *m* would have the same frequency.**

- **Rotation lifts this degeneracy, giving rise to "rotational splittings" of the modes:**

$$D_{nlm} = \frac{\nu_{nlm} - \nu_{nl-m}}{2m} = \int_0^1 \int_0^1 dr\ d\cos\theta\ K_{nlm}(r,\theta)\Omega(r,\theta)$$

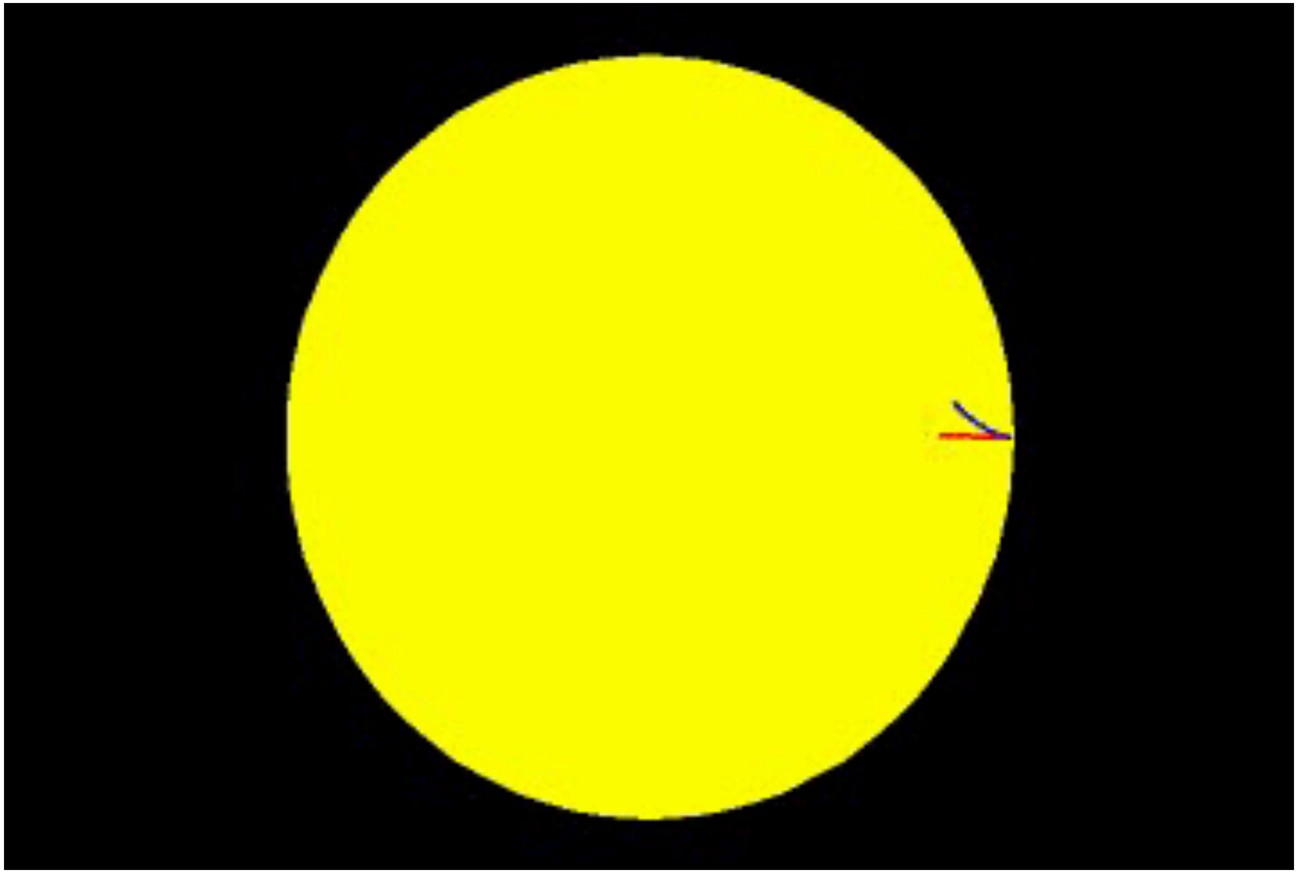
$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{max}} a_j(n,l)\mathcal{P}_j^{(l)}(m).$$

- **The central frequency is used to determine the spherically symmetric structure of the Sun.**

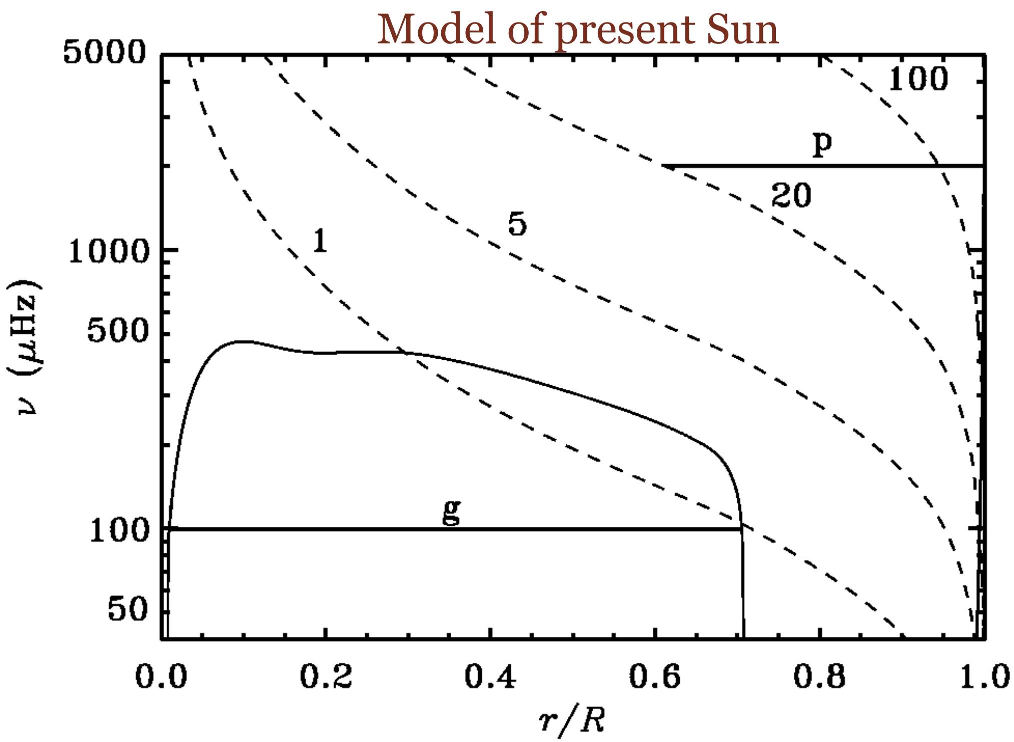
- **Odd order splitting coefficients (corresponding to the symmetric part of the splitting coefficients) are used to determine the rotation rate inside the Sun.**

- **Even-order splitting coefficients (corresponding to the antisymmetric part of the splitting coefficients) are used to determine asphericity.**

Different modes penetrate to different depths



Describing the modes



$$\frac{d^2\xi_r}{dr^2} \simeq -\frac{\omega^2}{c^2}\left(\frac{S_l^2}{\omega^2} - 1\right)\left(\frac{N^2}{\omega^2} - 1\right)\xi_r$$

Eigenfunction oscillates as function of *r* when

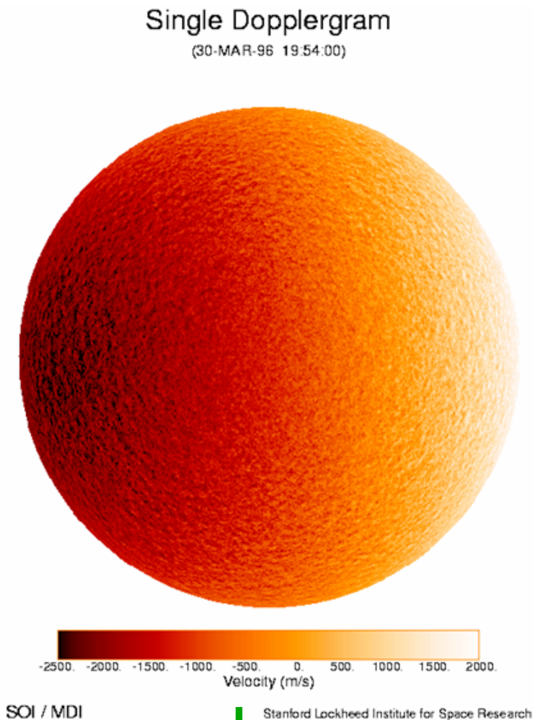
$\omega^2 > S_l^2, N^2$ **p modes**

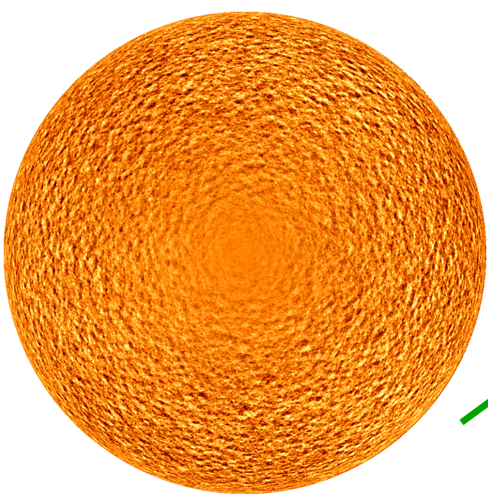
$\omega^2 < S_l^2, N^2$ **g modes**

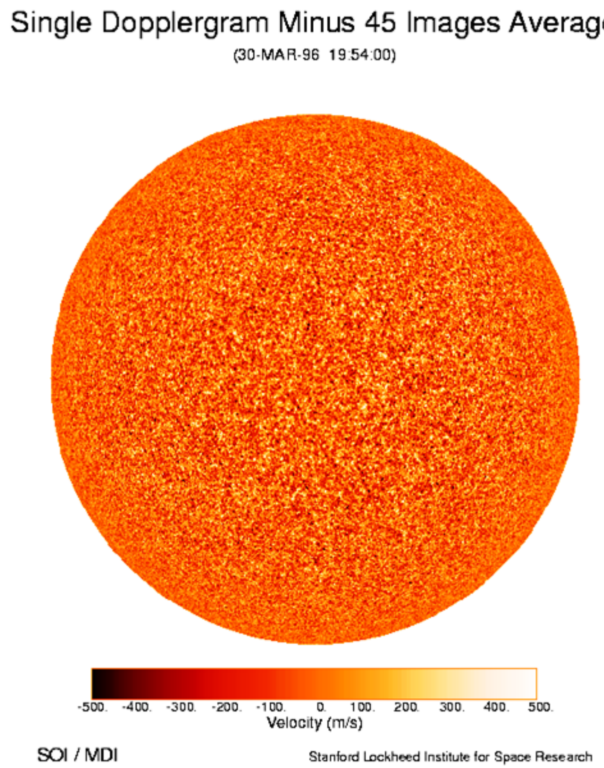
$$S_l^2 = \frac{l(l+1)c^2}{r^2}$$

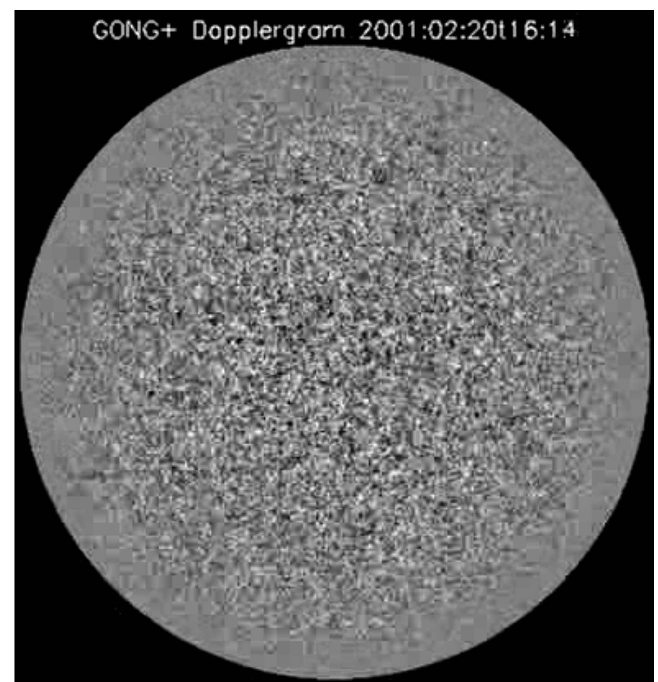
$$N^2 = g\left(\frac{1}{\Gamma_1}\frac{d\ln p}{dr} - \frac{d\ln \rho}{dr}\right) \simeq \frac{g^2\rho}{p}(\nabla_{ad} - \nabla + \nabla_\mu$$

What do solar observations look like?

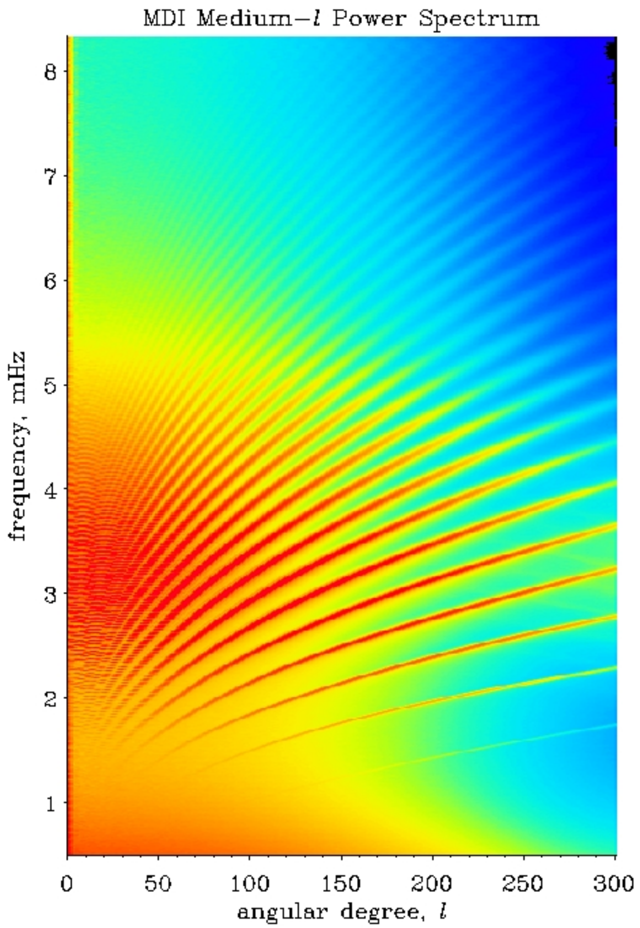




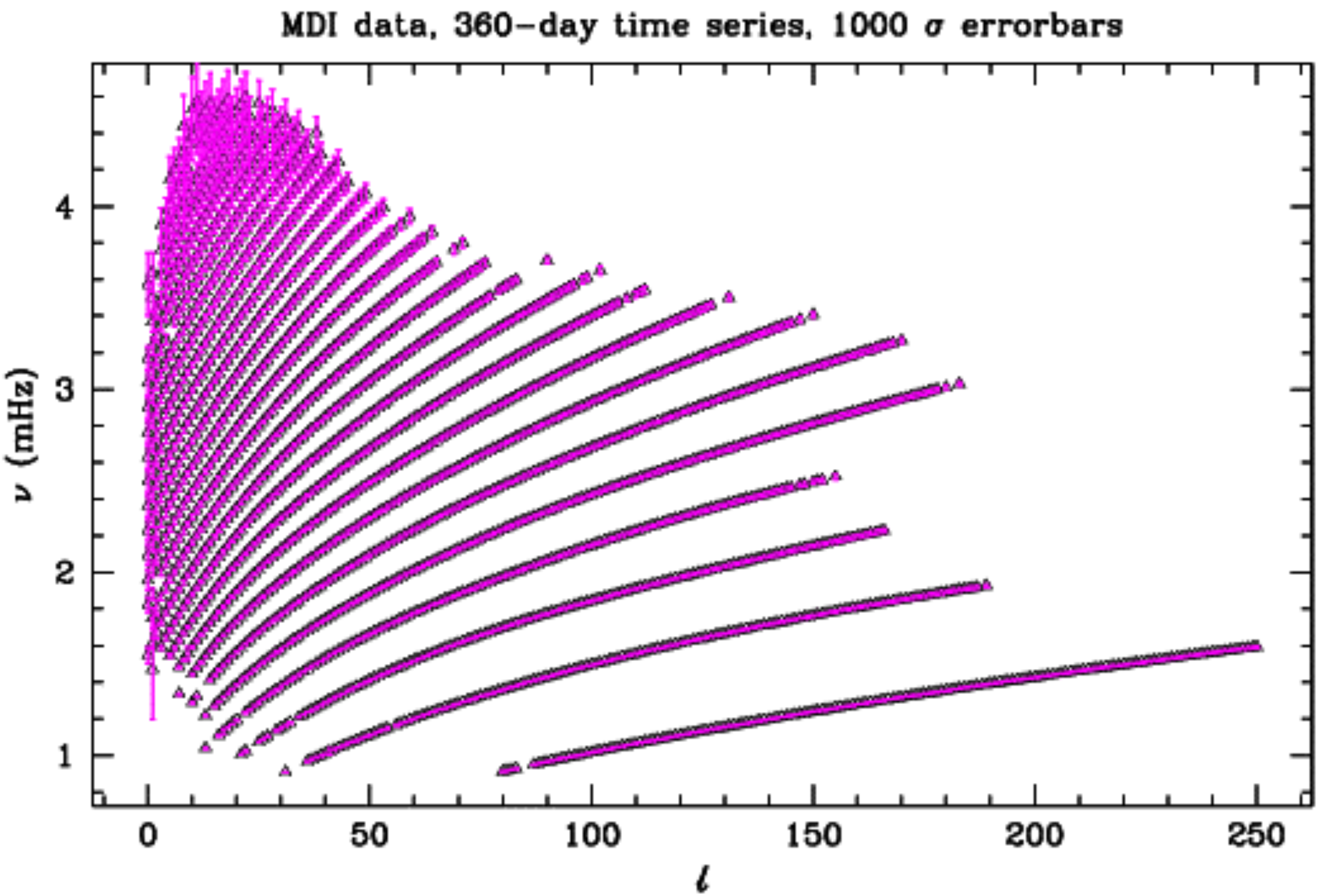




A Solar Power Spectrum



A sample of helioseismic data



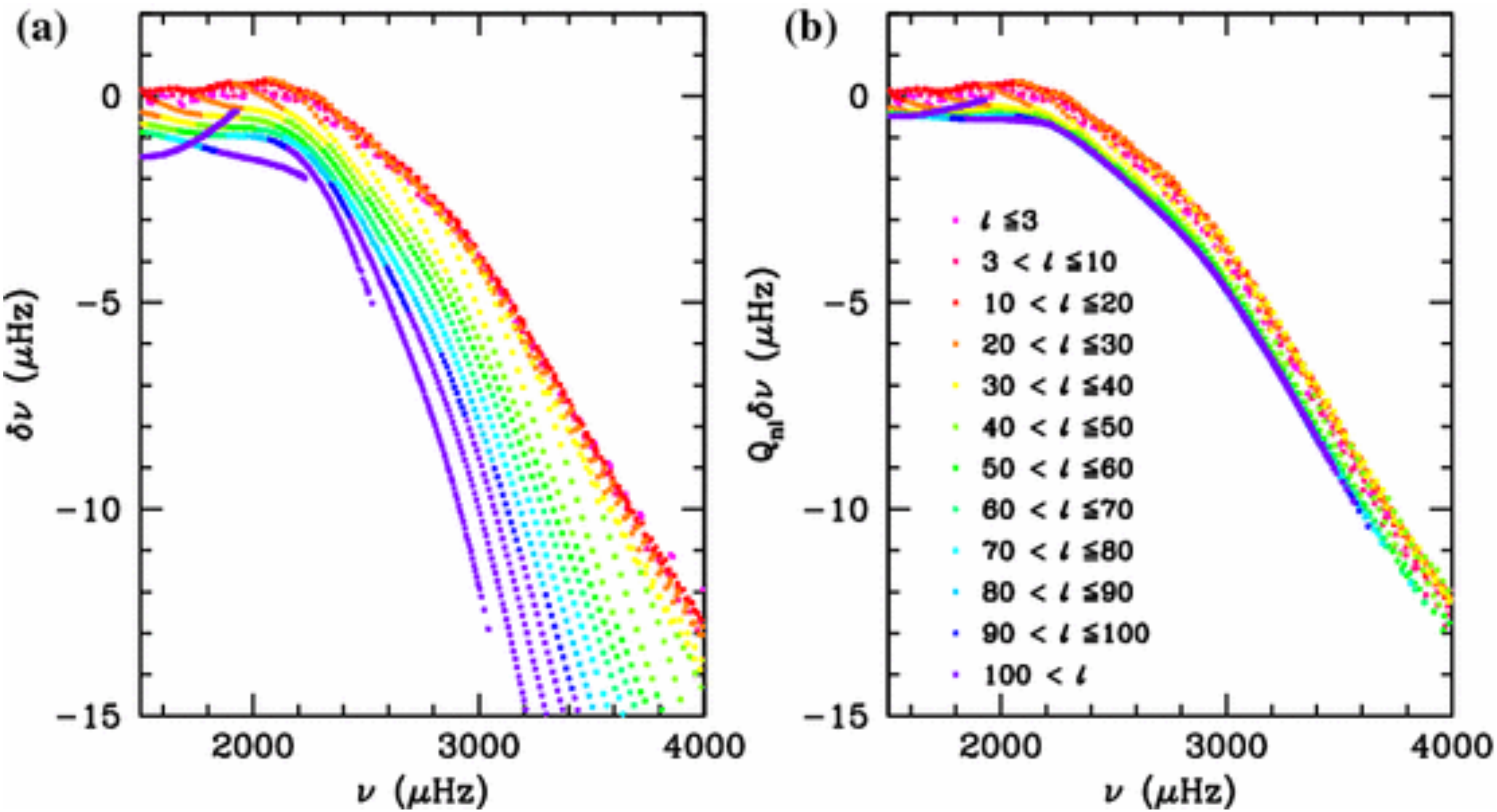
What have we learned?

- Solar structure
- Solar dynamics
  - Physics!

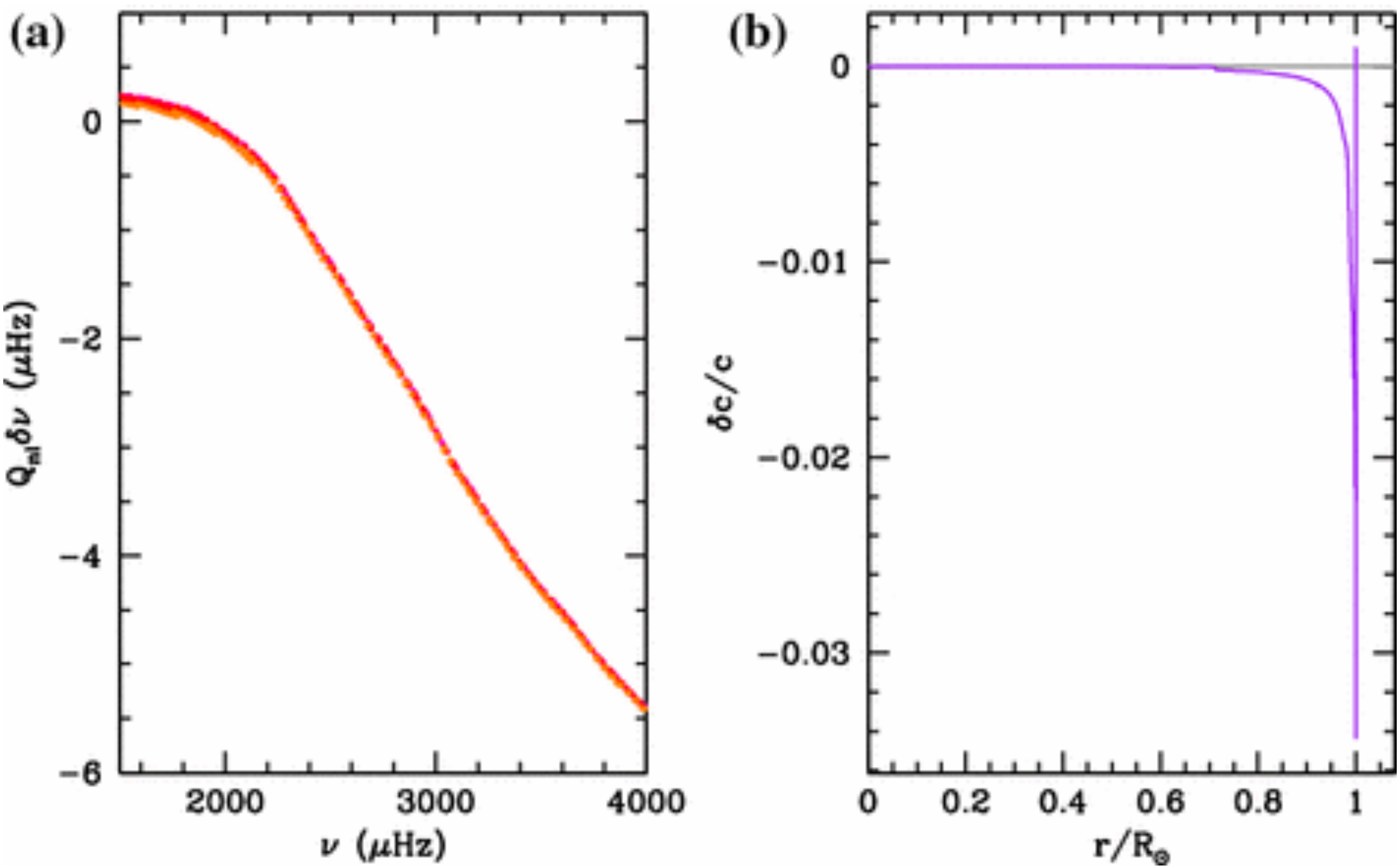
and

- Solar-cycle related changes

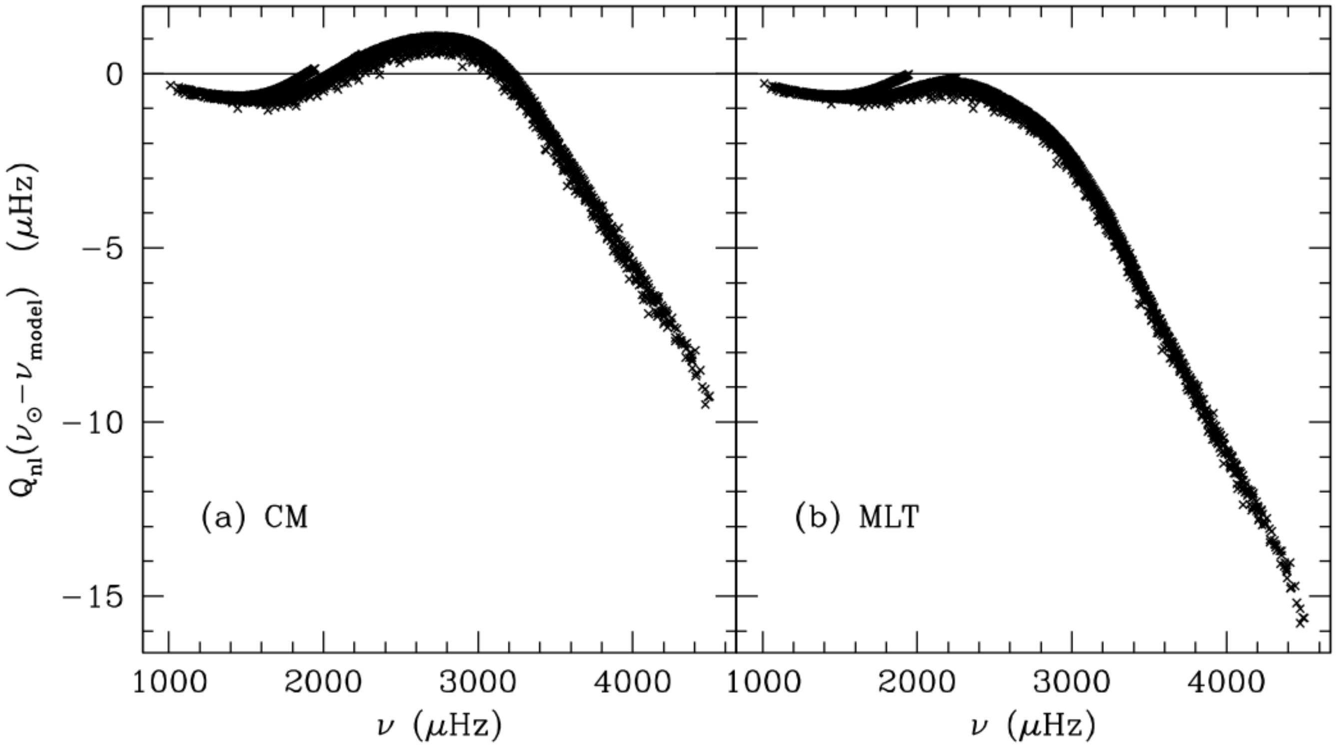
Comparing frequencies does not work



Changes in surface physics changes frequencies



Complication: the Surface Term



We therefore resort to inversions

The inverse problem for structure

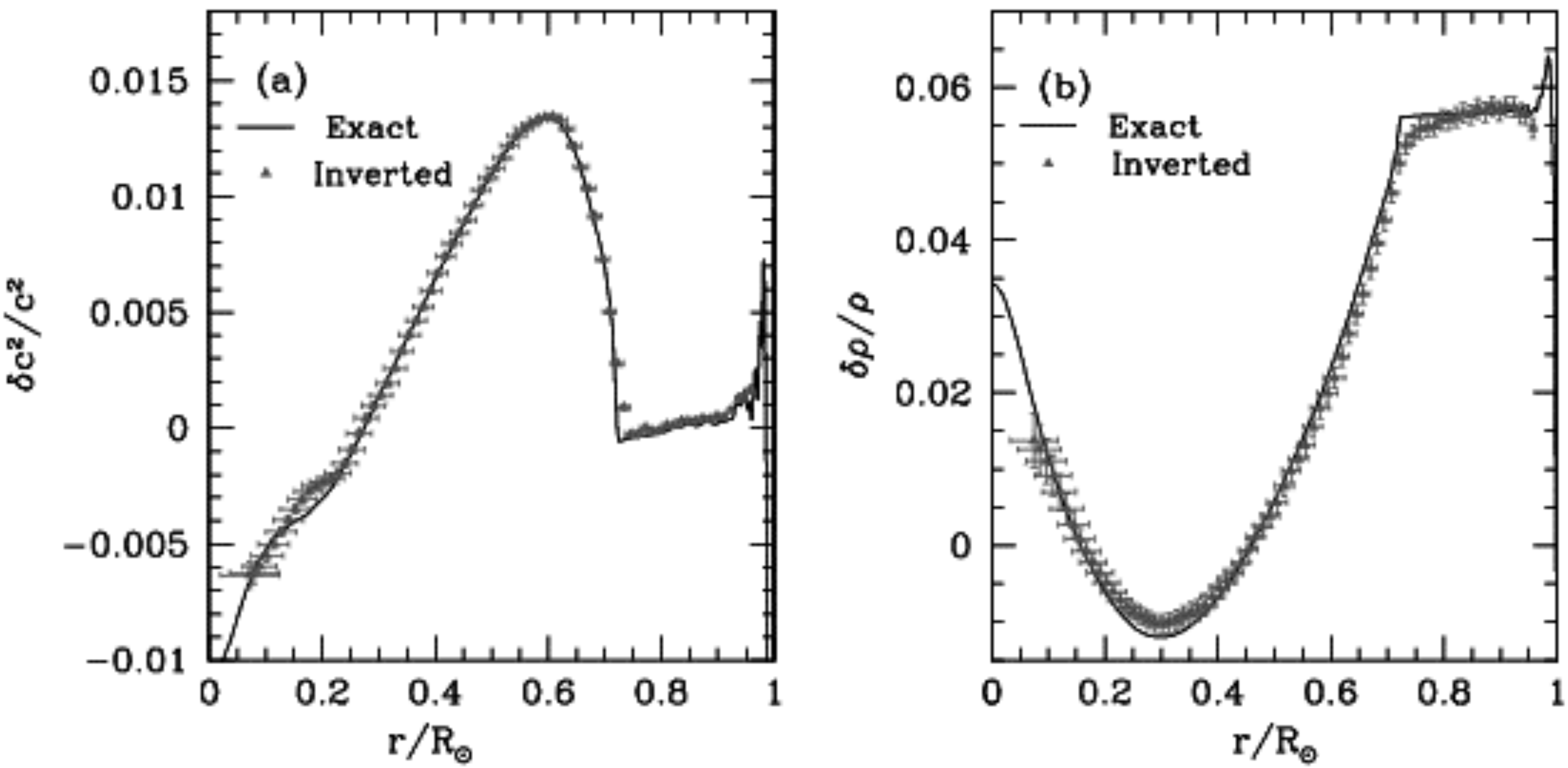
Relationship between structure and frequencies:

$$-\omega^2 \rho \vec{\xi} = \nabla \left(c^2 \rho \nabla \cdot \vec{\xi} + \nabla p \cdot \vec{\xi} \right) - \vec{g} \nabla \cdot (\rho \vec{\xi}) - G \rho \nabla \left(\int_v \frac{\nabla \cdot (\rho \vec{\xi}) d^3 \vec{r'}}{|\vec{r} - \vec{r'}|} \right)$$

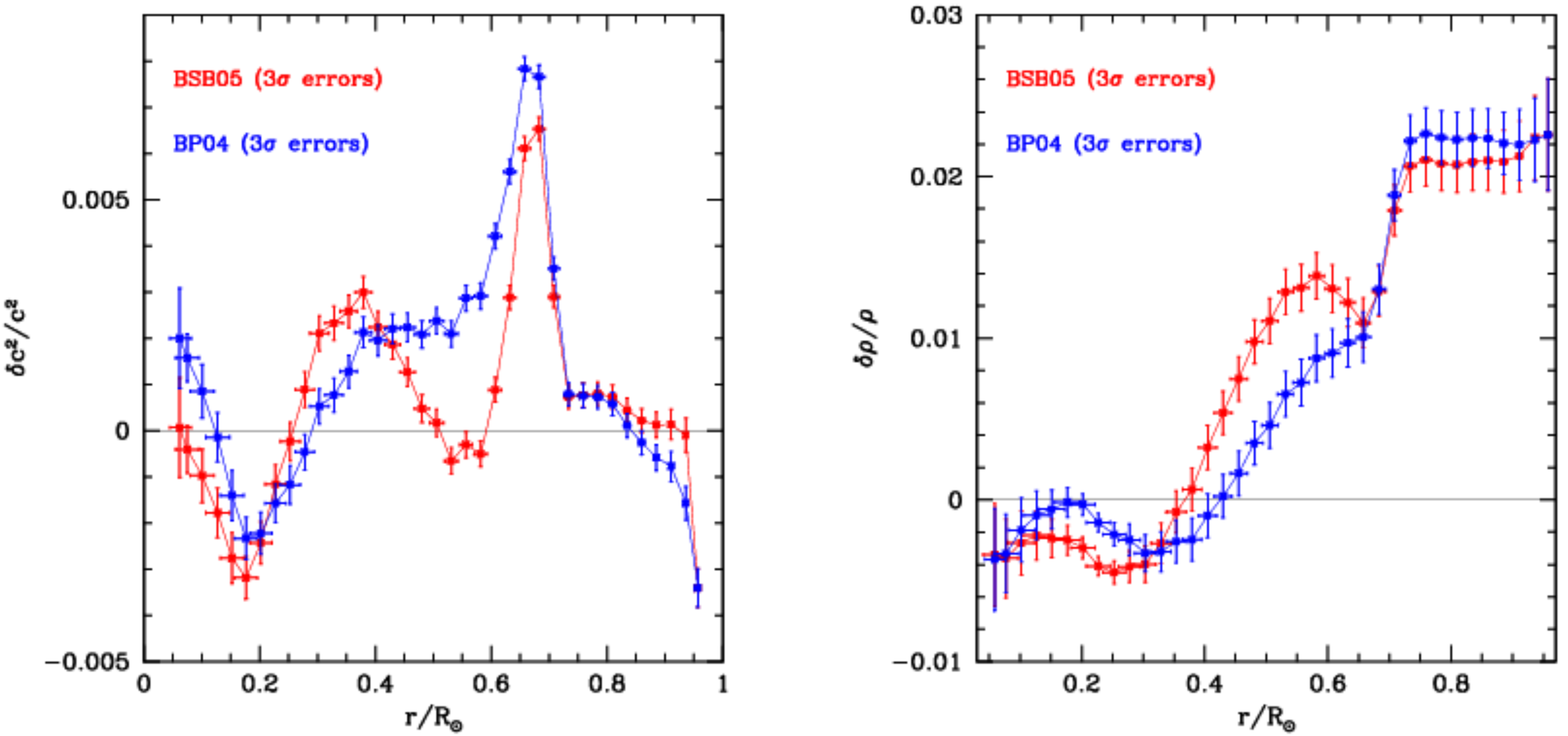
A Hermitian Eigenvalue problem, threfore use the variational principle:

$$\frac{\delta \omega_i}{\omega_i} = \int K^i_{c^2,\rho}(r) \frac{\delta c^2}{c^2} dr + \int K^i_{\rho,c^2}(r) \frac{\delta \rho}{\rho} dr + \frac{F_{\rm surf}(\omega_i)}{I_i}$$

We can actually invert for structure differences



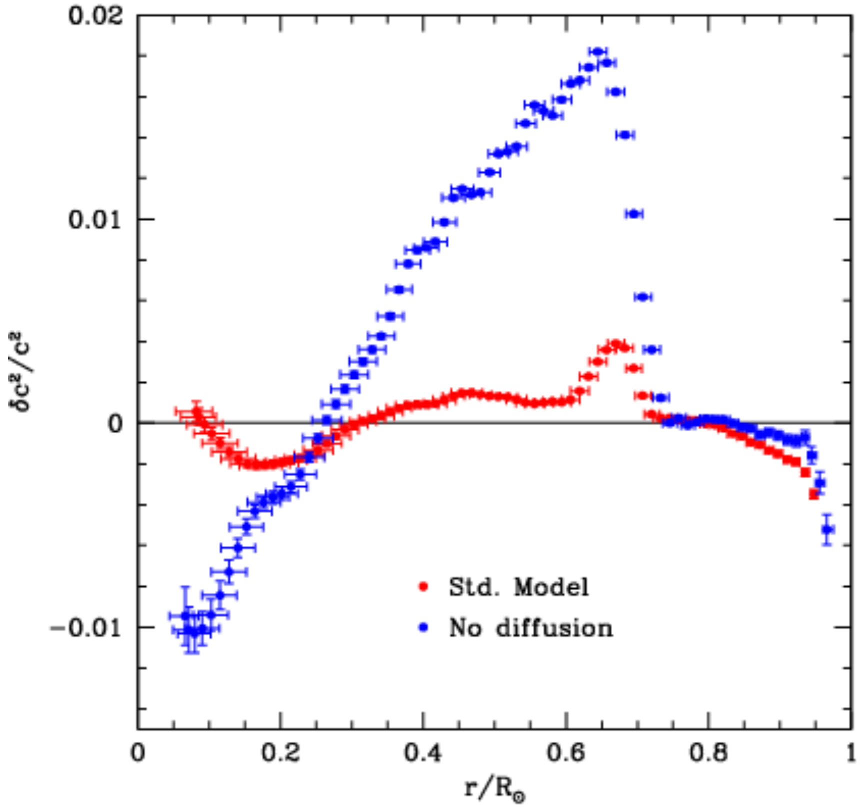
Inversion results showing differences between models and the Sun

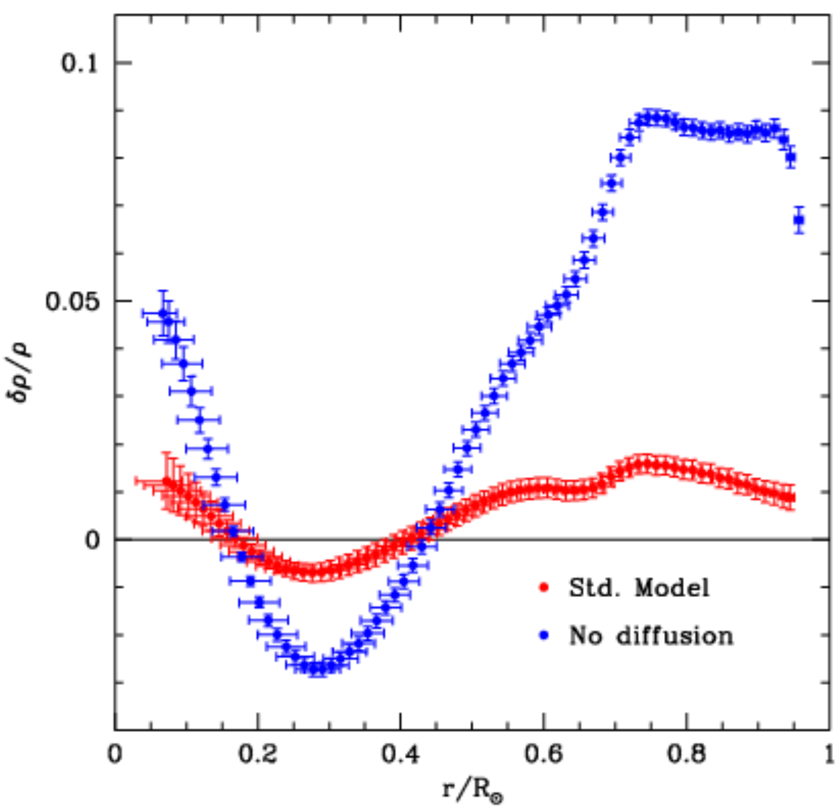


Models from Bahcall, Basu & Serenelli (2005)

Inversions from Basu et al. (2009)

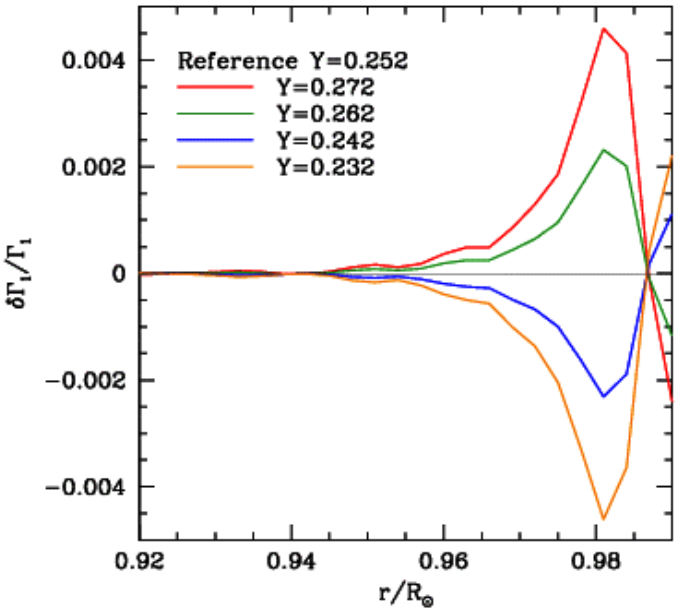
We can determine which physical processes are important



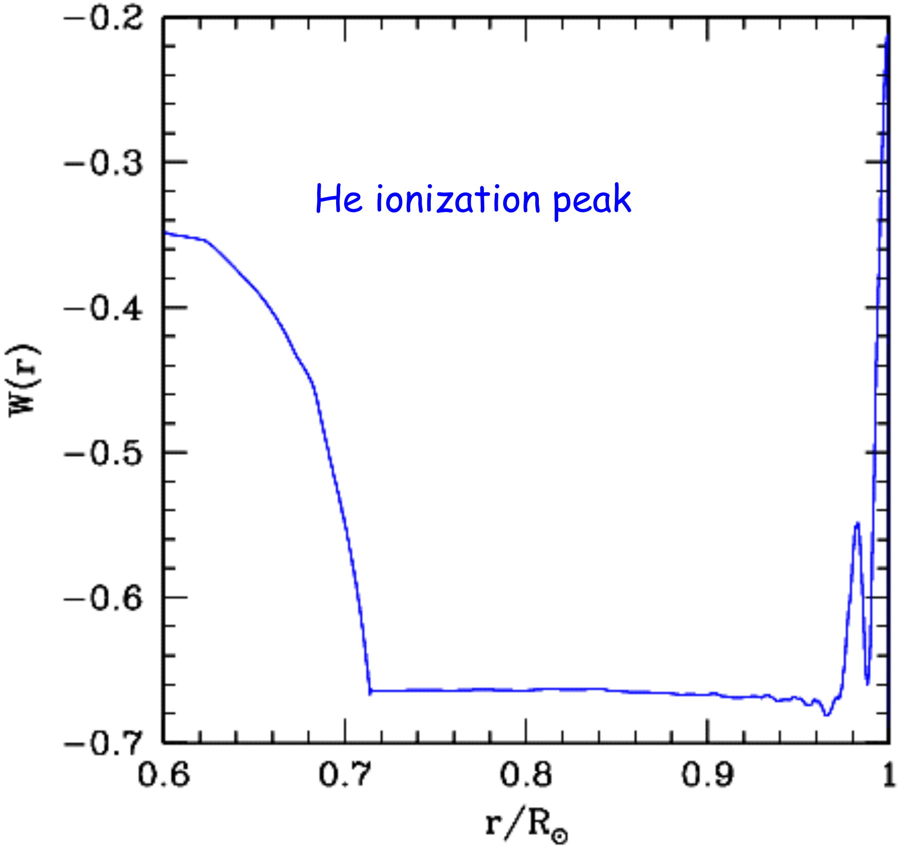


Models from Basu, Pinsonneault & Bahcall 2000

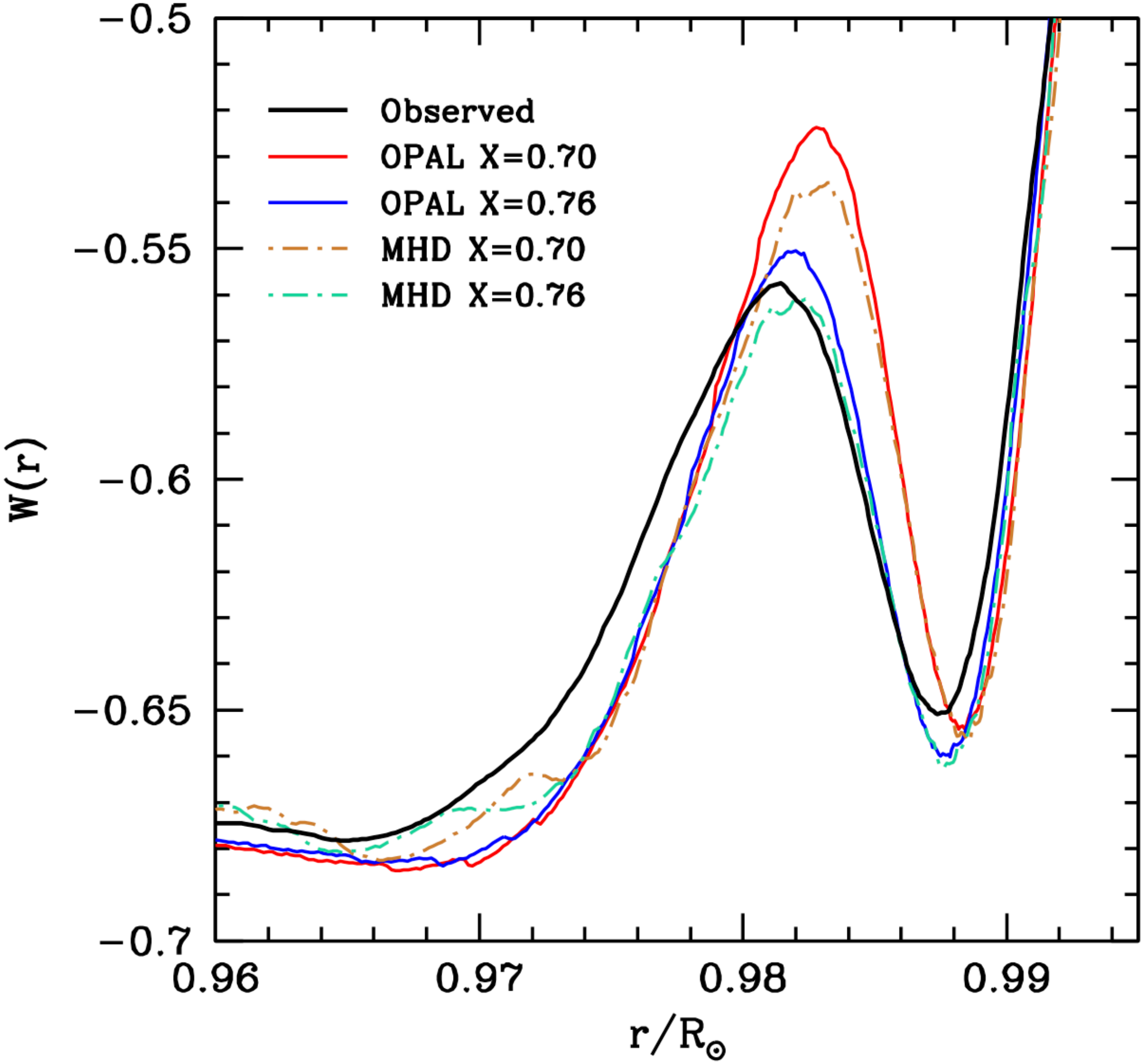
We can determine the helium abundance of the solar convection zone



$$W(r) = \frac{r^2}{Gm}\frac{\mathrm{d}c^2}{\mathrm{d}r}$$

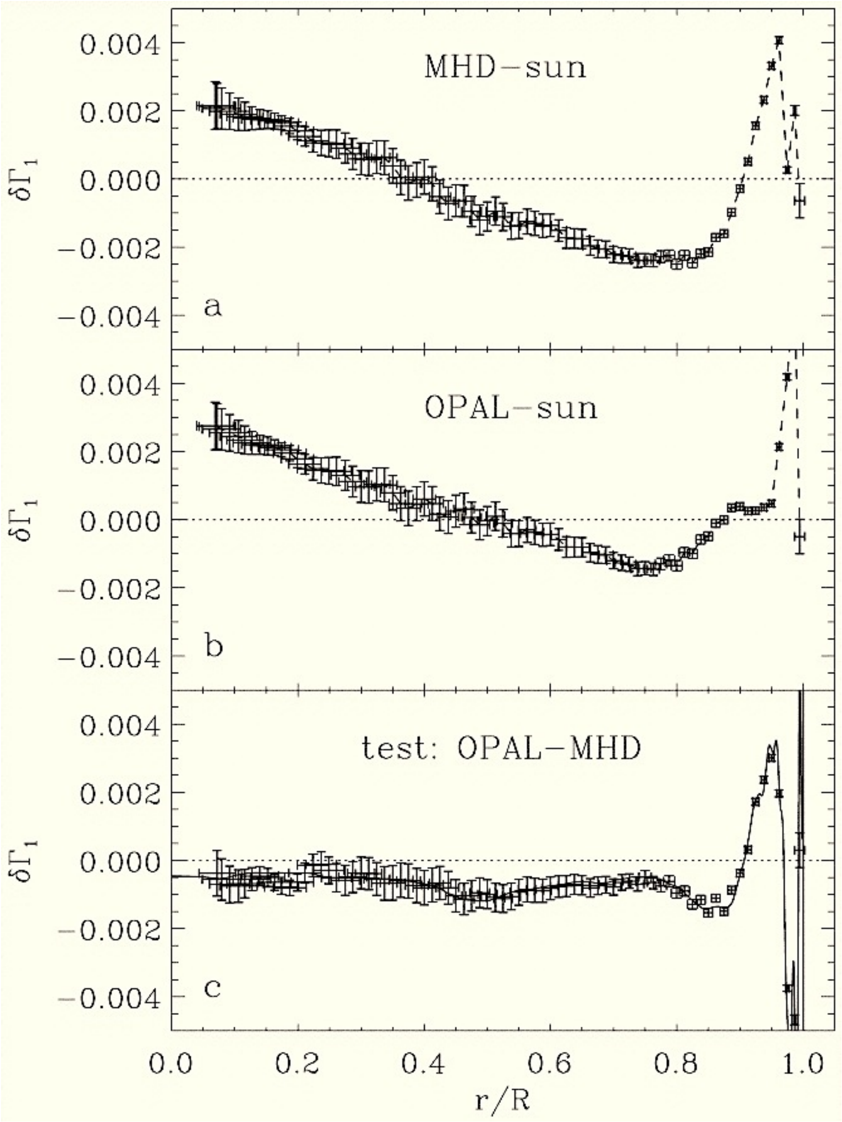


$$c^2 = \Gamma_1 \frac{p}{\rho}$$



Observed
OPAL X=0.70
OPAL X=0.76
MHD X=0.70
MHD X=0.76
W(r)
r/R⊙
−0.5
−0.55
−0.6
−0.65
−0.7
0.96
0.97
0.98
0.99

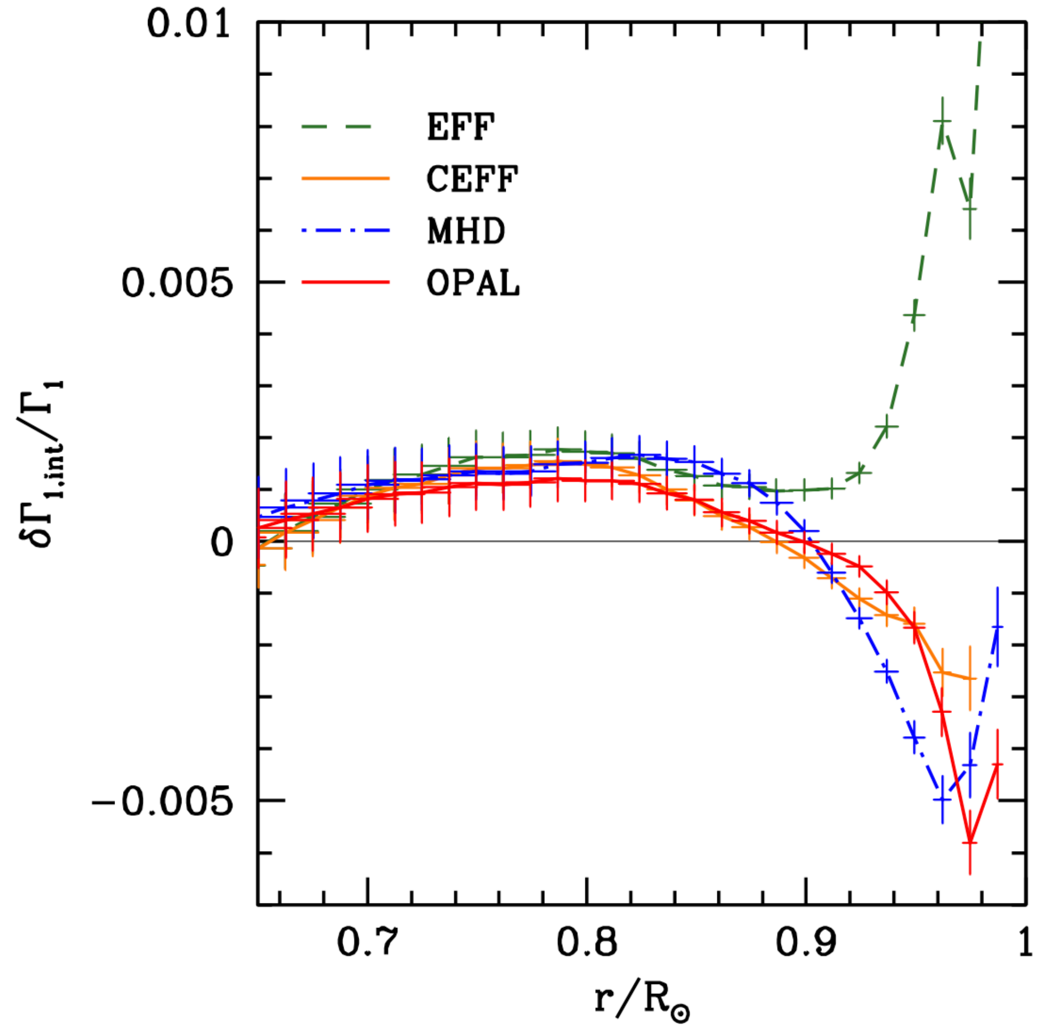
More physics: The equation of state



From Elliott & Kosovichev 1998

Can we eliminate the composition term?

$$\frac{\delta\Gamma_1}{\Gamma_1} = \left(\frac{\partial \ln \Gamma_1}{\partial Y}\right)_{P,\rho} \delta Y + \left(\frac{\partial \ln \Gamma_1}{\partial \ln P}\right)_{\rho,Y} \frac{\delta P}{P} + \left(\frac{\partial \ln \Gamma_1}{\partial \ln \rho}\right)_{P,Y} \frac{\delta\rho}{\rho} \quad \left(\frac{\delta\Gamma_1}{\Gamma_1}\right)_{\text{int}}$$

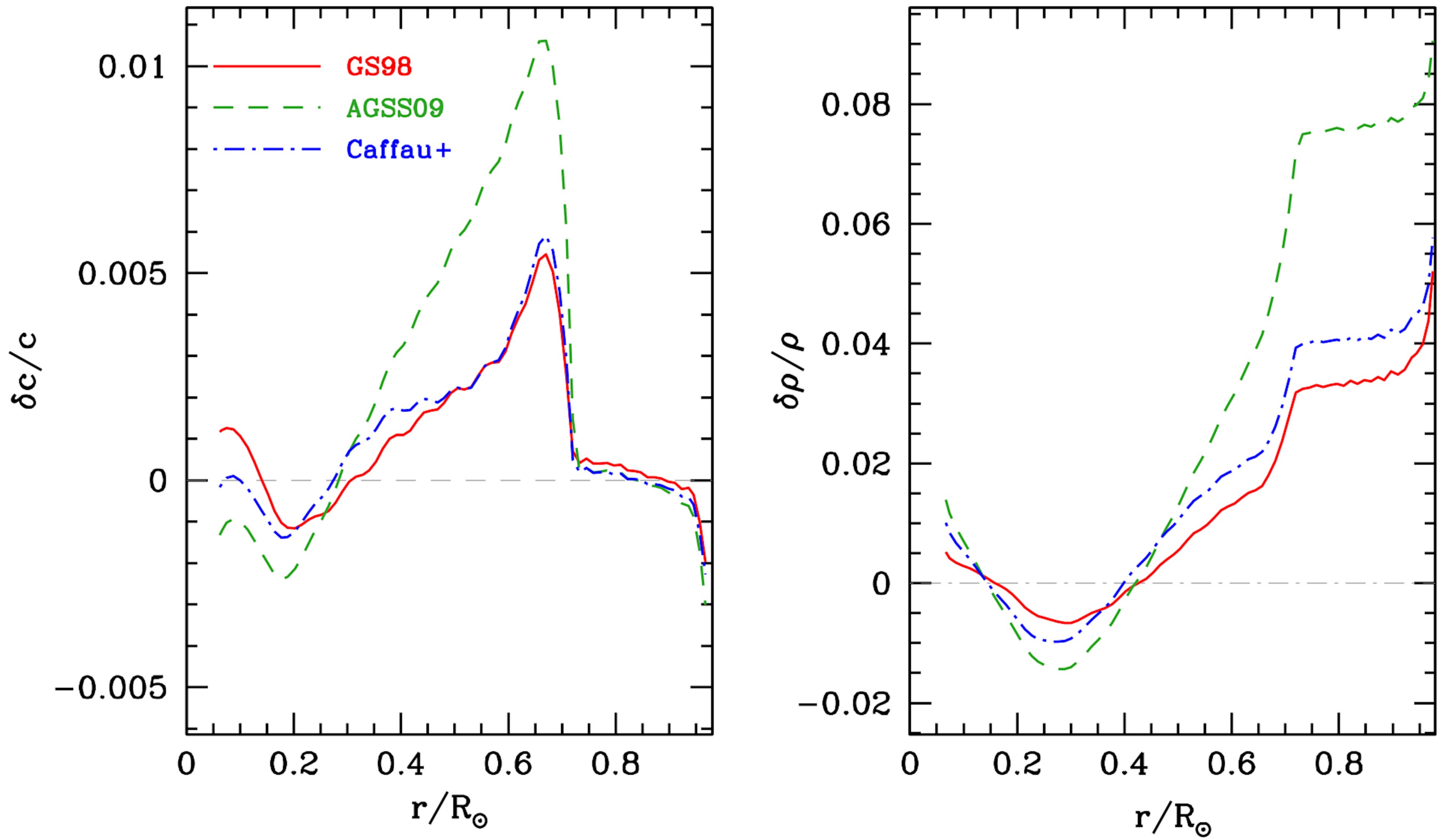


Results from Basu & Christensen-Dalsgaard 1997

The current headache: What is the metallicity of the Sun?

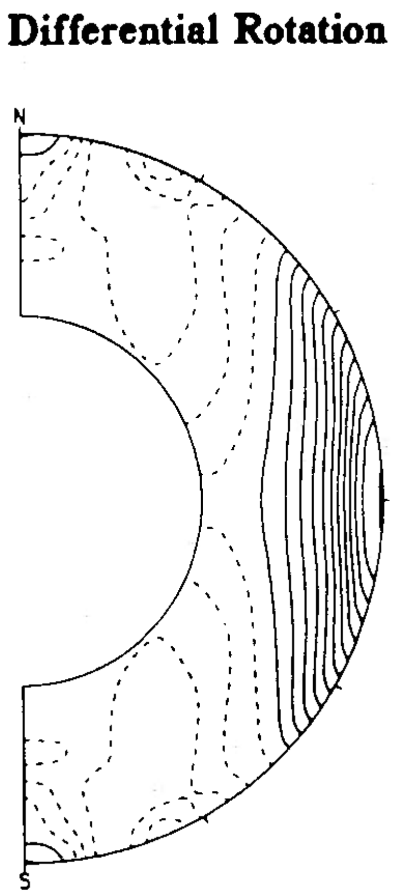
Element	GS98	AGS05	Caffau et al. (2010)	Asplund et al. (2009)
C	8.52±0.06	8.39±0.05	8.50±0.06	8.43±0.05
N	7.92±0.06	7.78±0.06	7.86±0.12	7.83±0.05
O	8.83±0.06	8.66±0.05	8.76±0.07	8.69±0.05
Fe	7.50±0.05	7.45±0.05	7.52±0.03	7.50±0.04
Z/X	**0.023**	**0.0165**	**0.0209**	**0.0181**

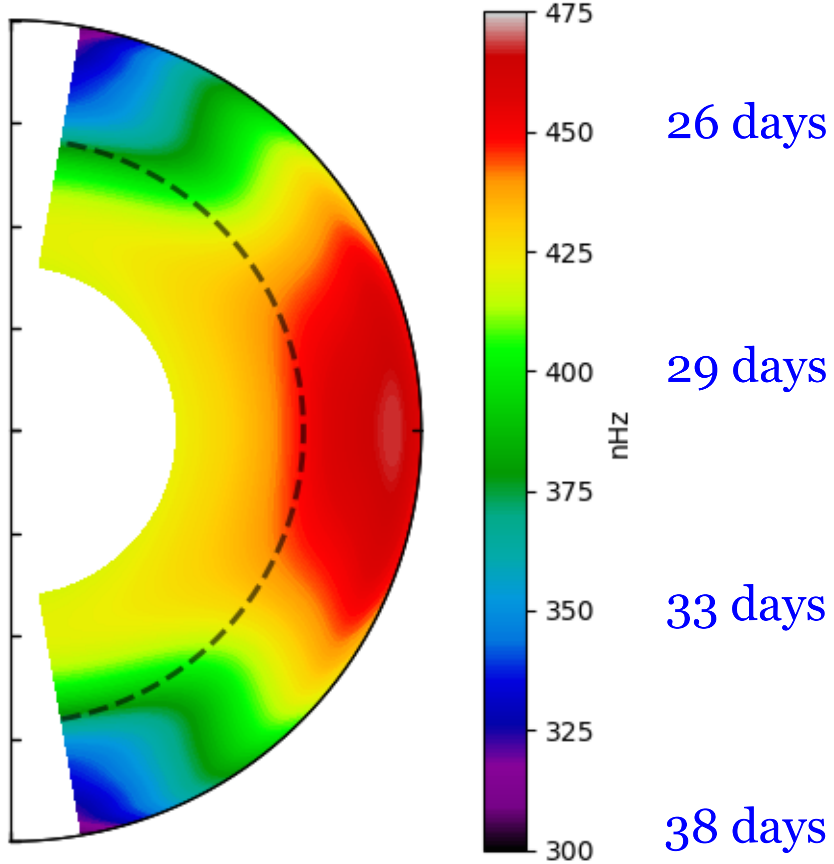
The sound speed and density profiles differ



What about
Solar dynamics?

The surprise of rotation

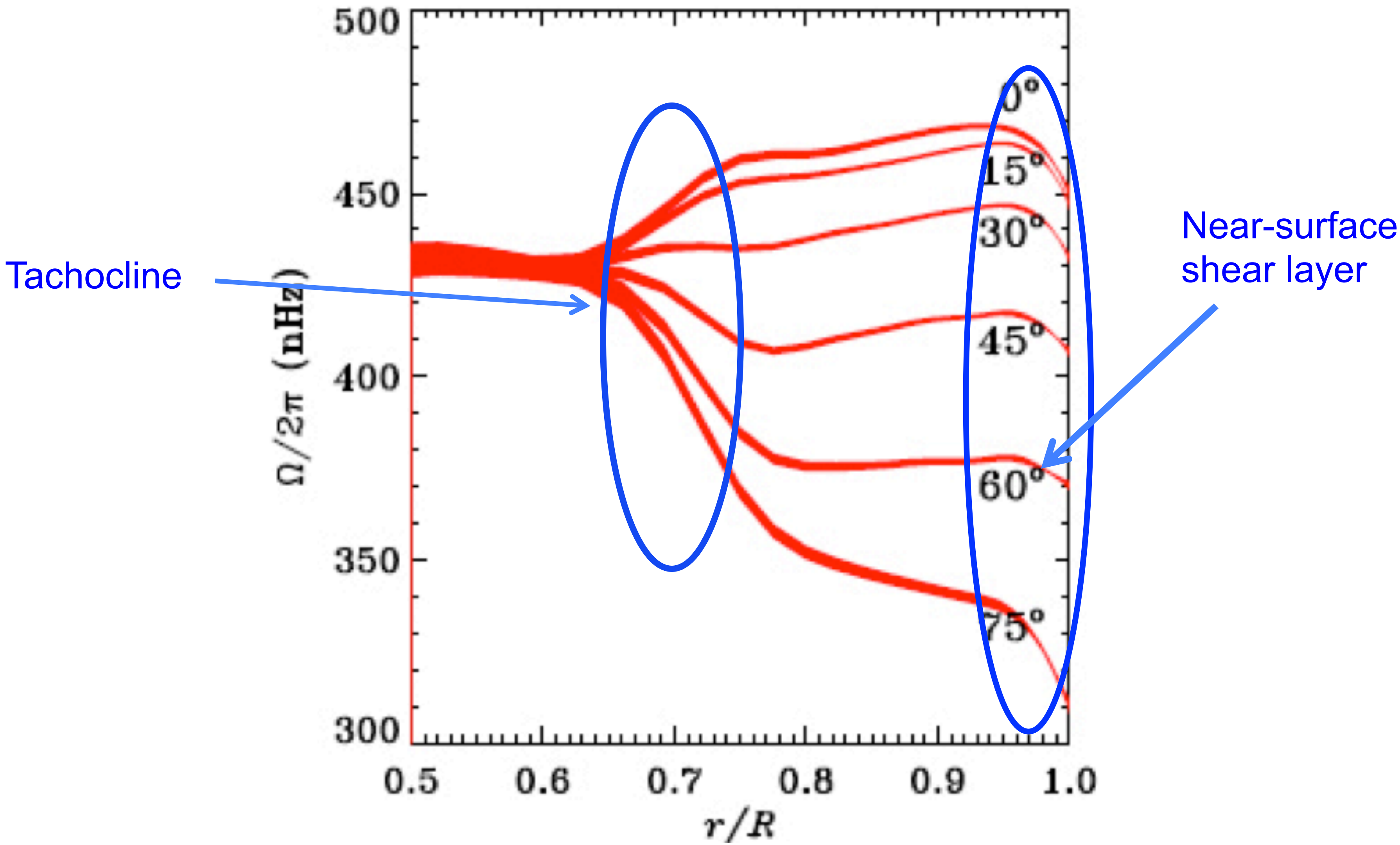




Glatzmaier 1985

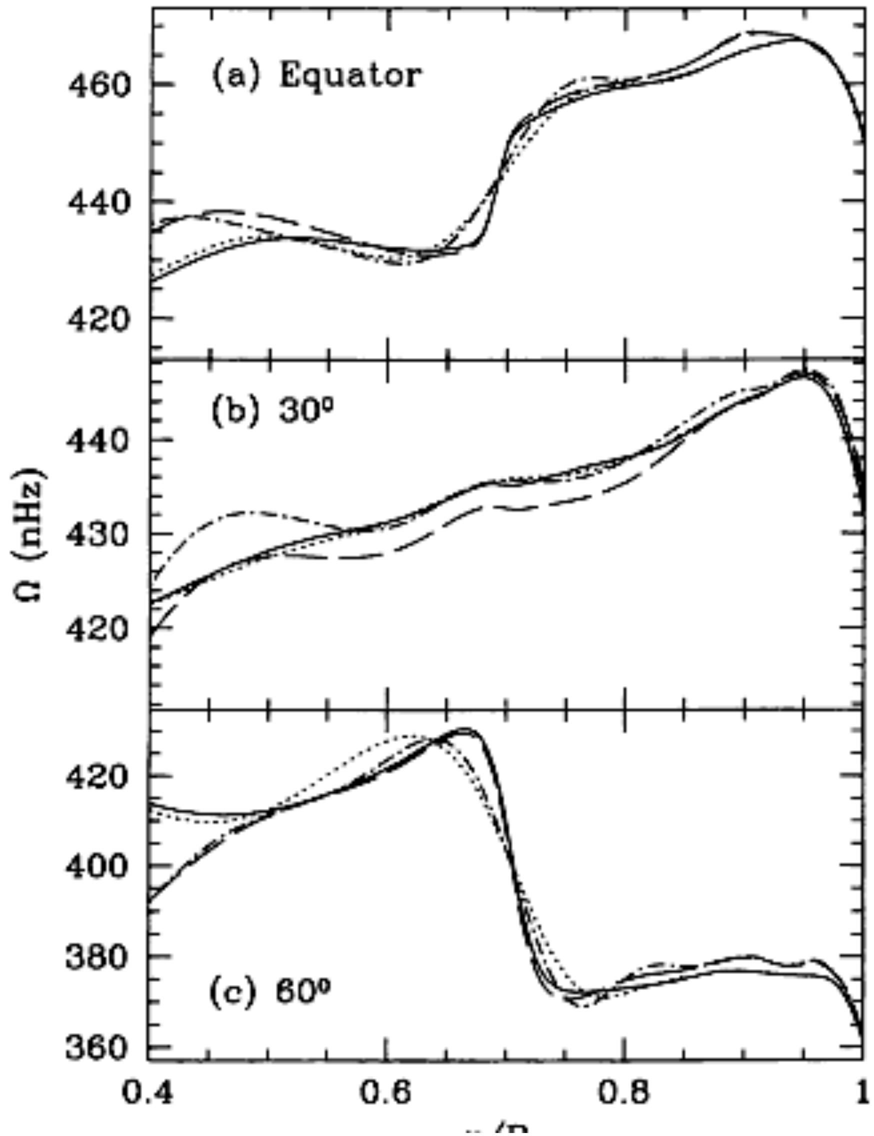
Antia & Basu 2018

A bit more detail



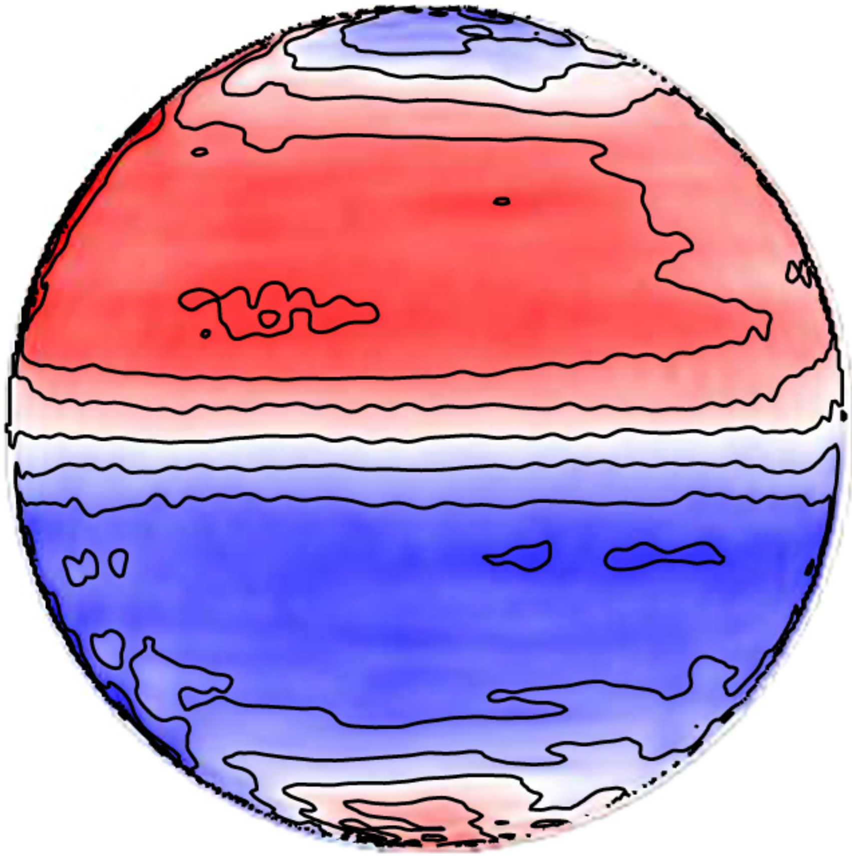
Howe 2015

The tachocline is extremely thin



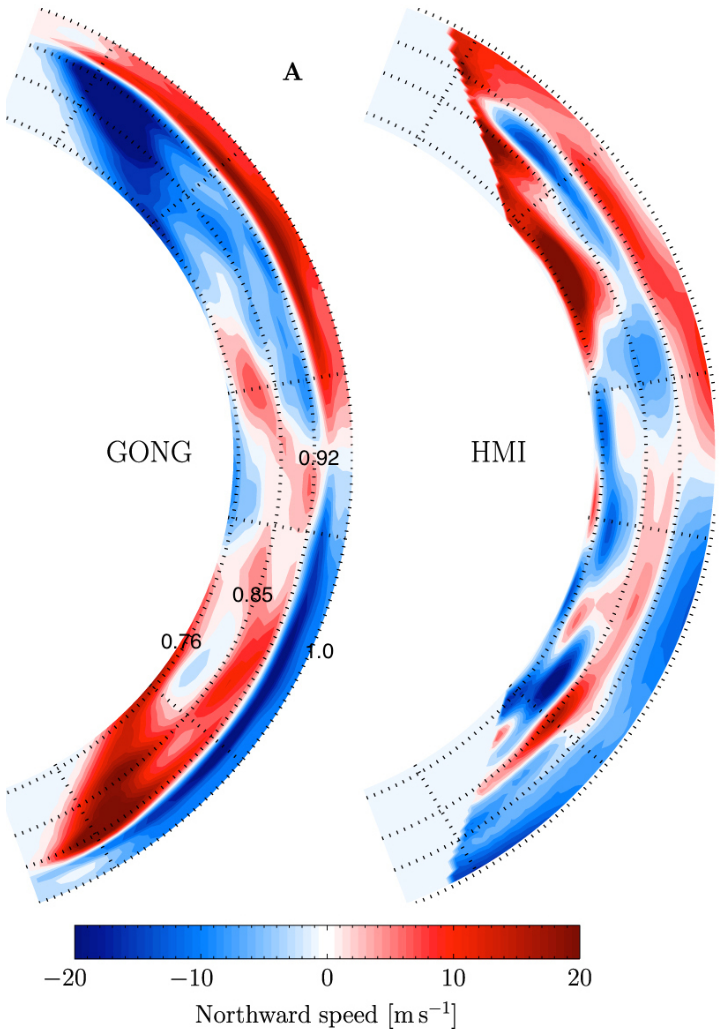
Antia, Basu & Chitre 1998

Meridional flows

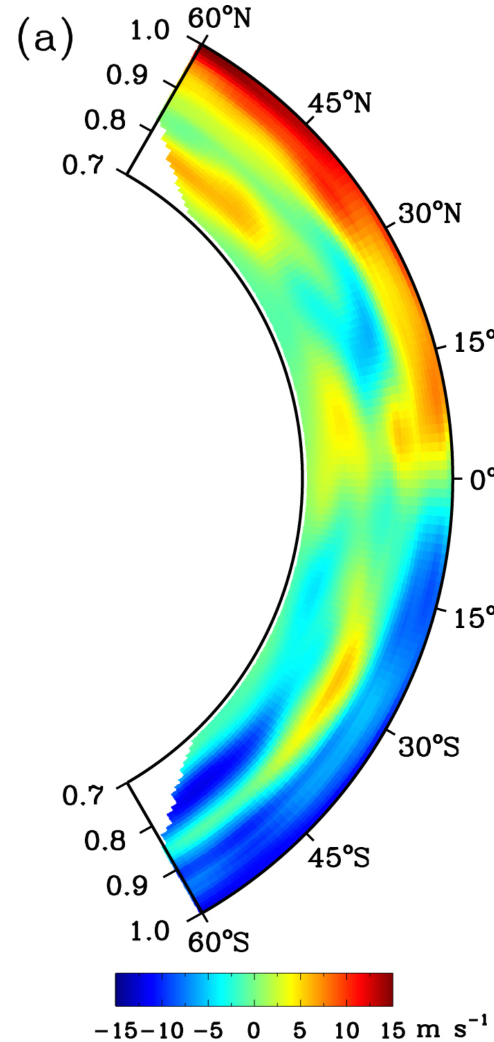


Bogart, Basu & Baldner 2015

Where do the meridional flows return?



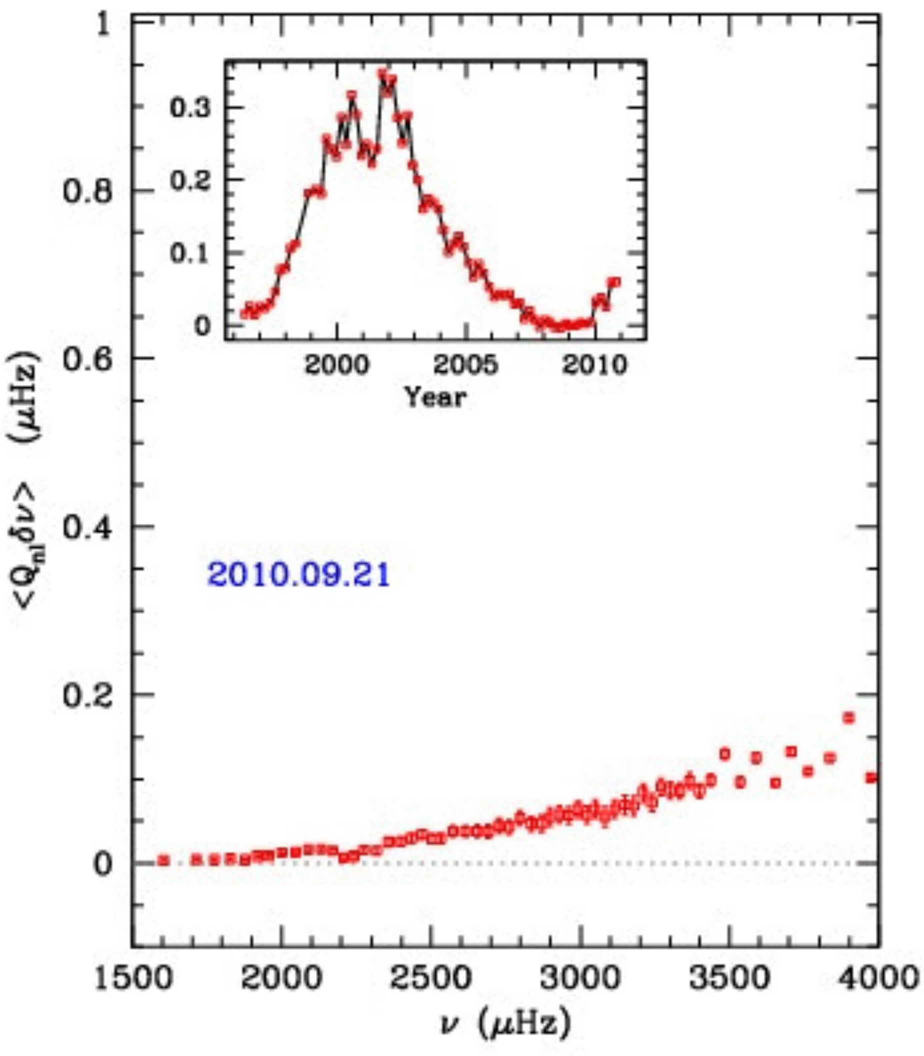
Jackiewicz et al. 2015



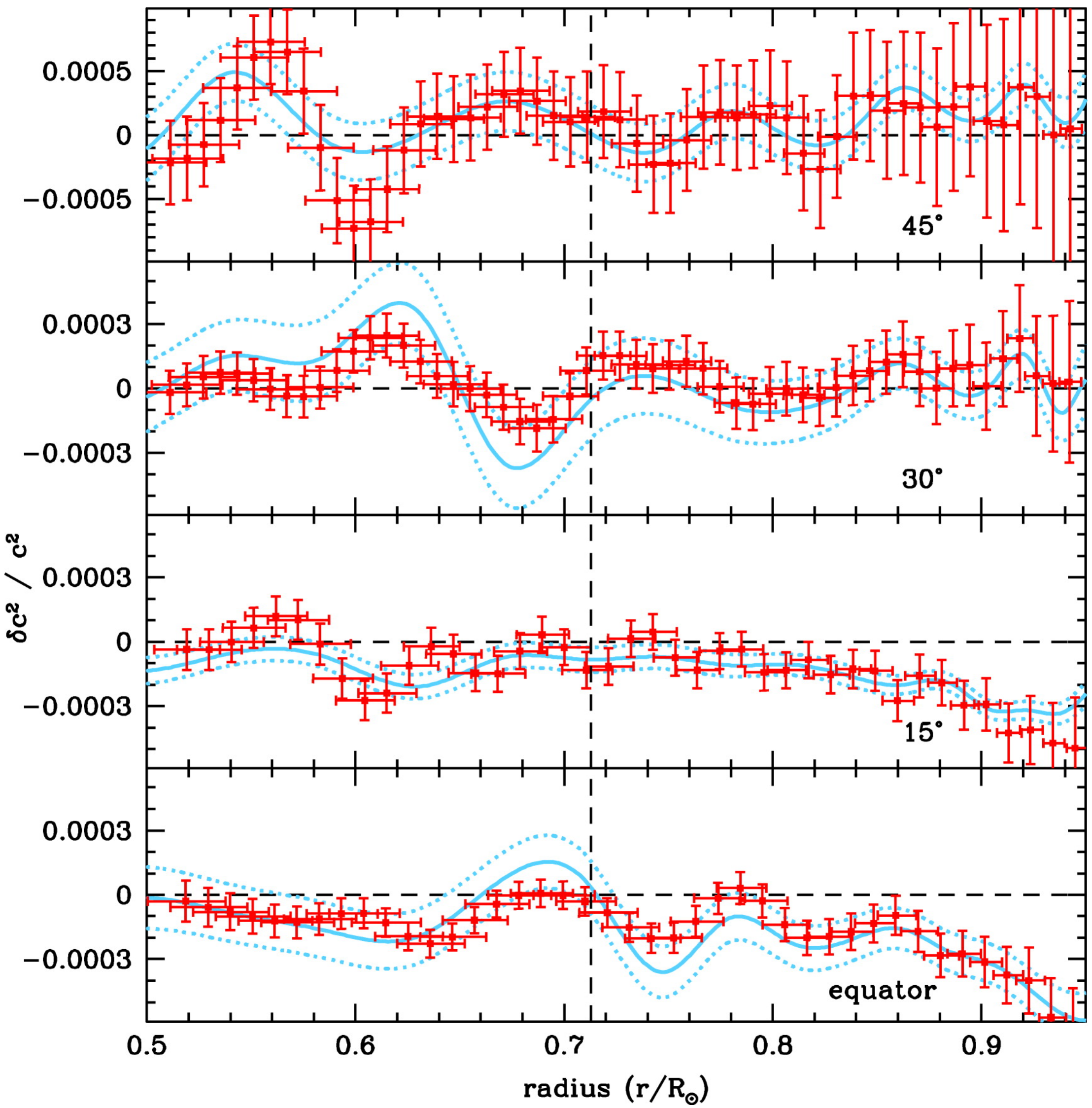
Chen & Zhao 2017

Solar activity related changes

Solar oscillation frequencies change with solar cycle

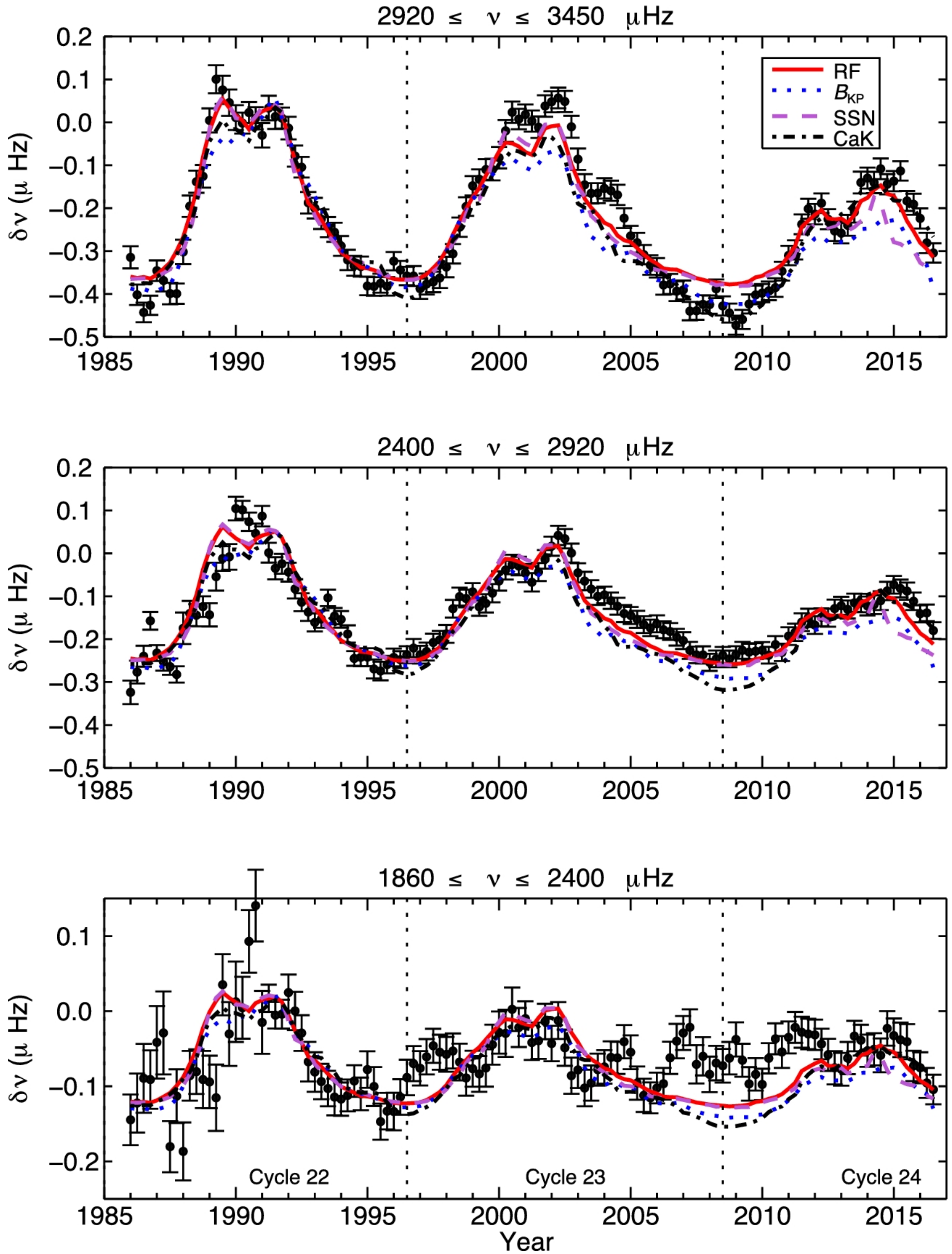


Solar sound-speed changes as a function of latitude



Baldner & Basu 2008

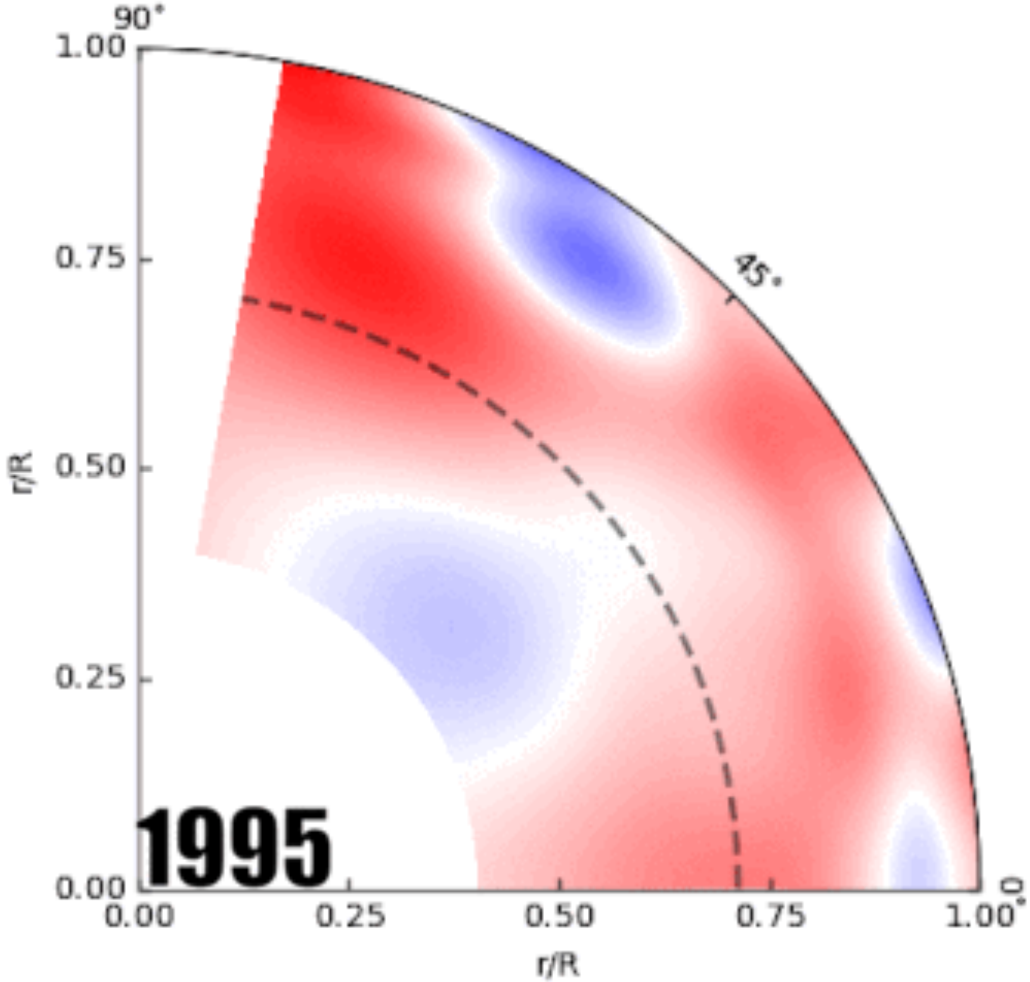
There are cycle-to-cycle differences

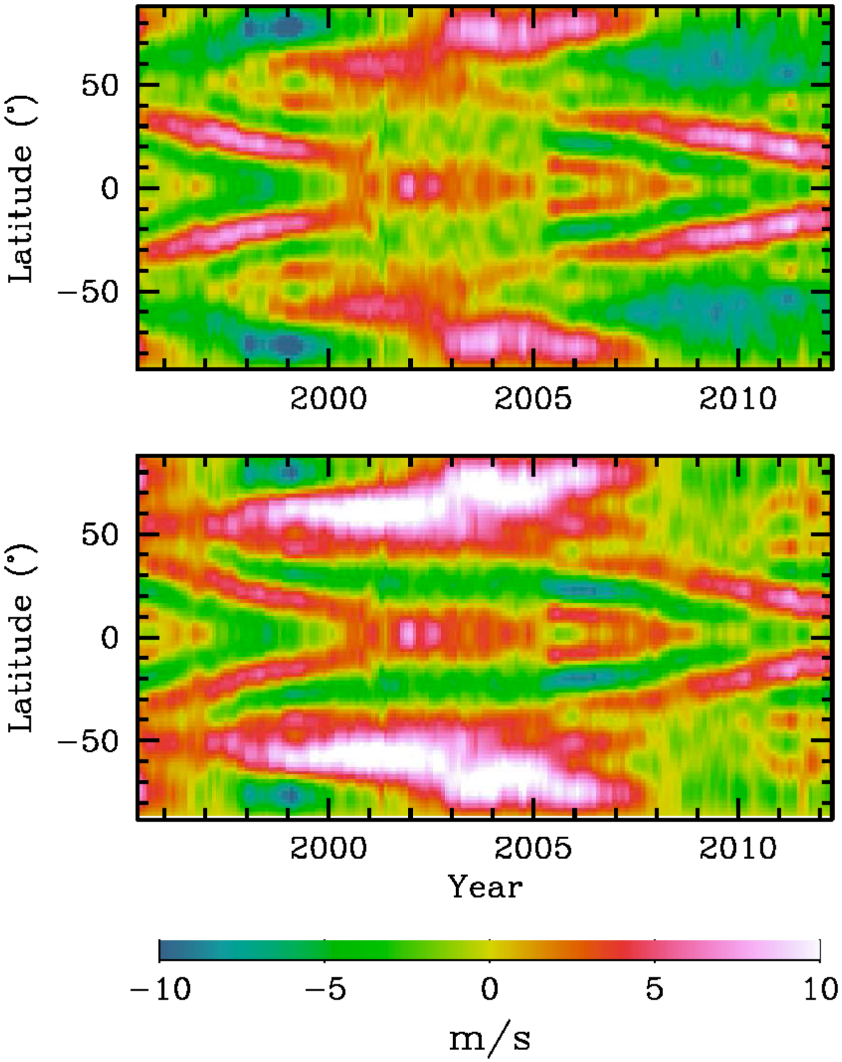


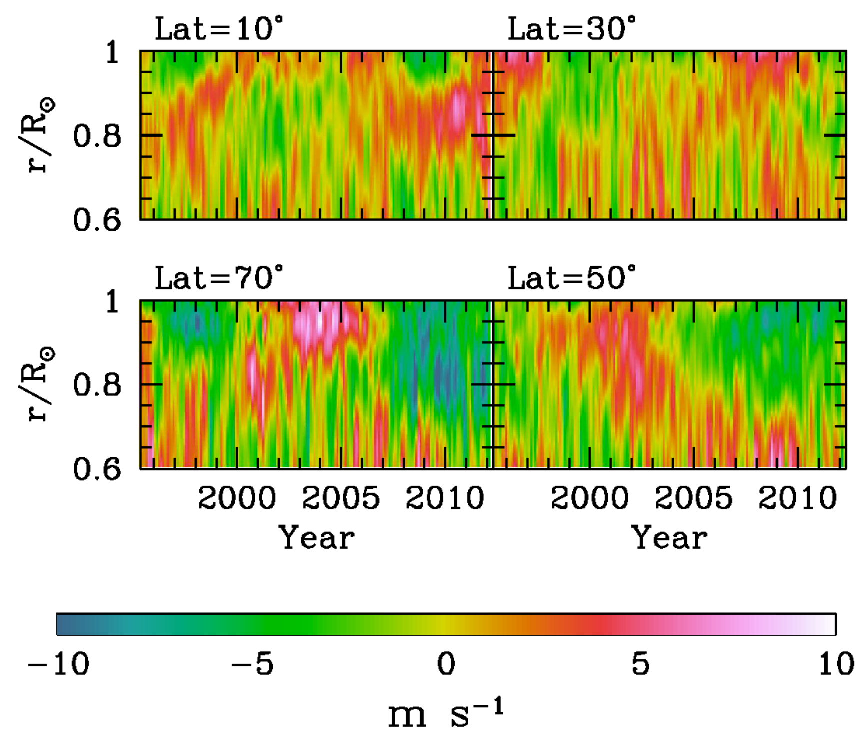
Howe et al. 2017

Changes in dynamics are much more interesting!

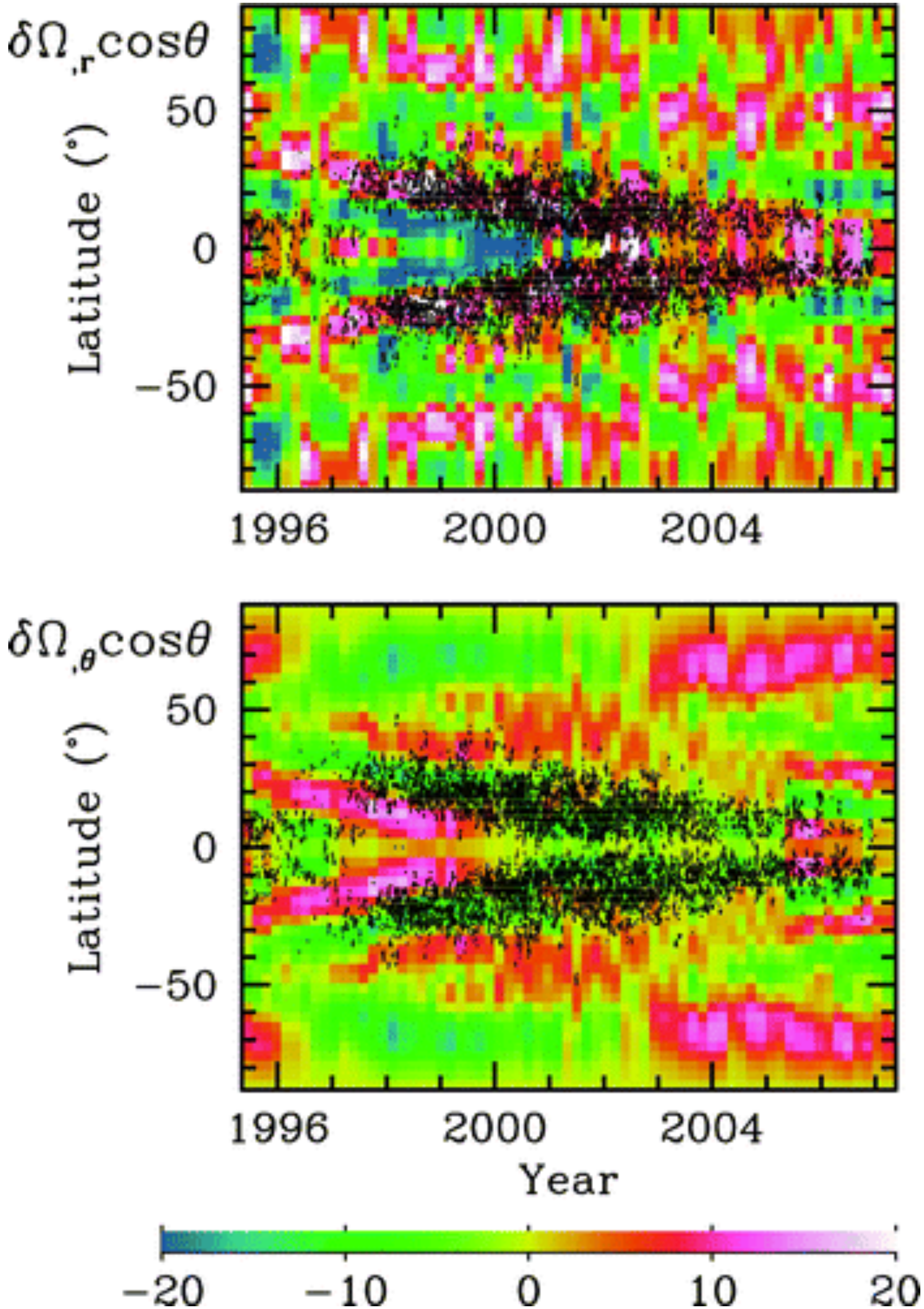
Changes in zonal flows



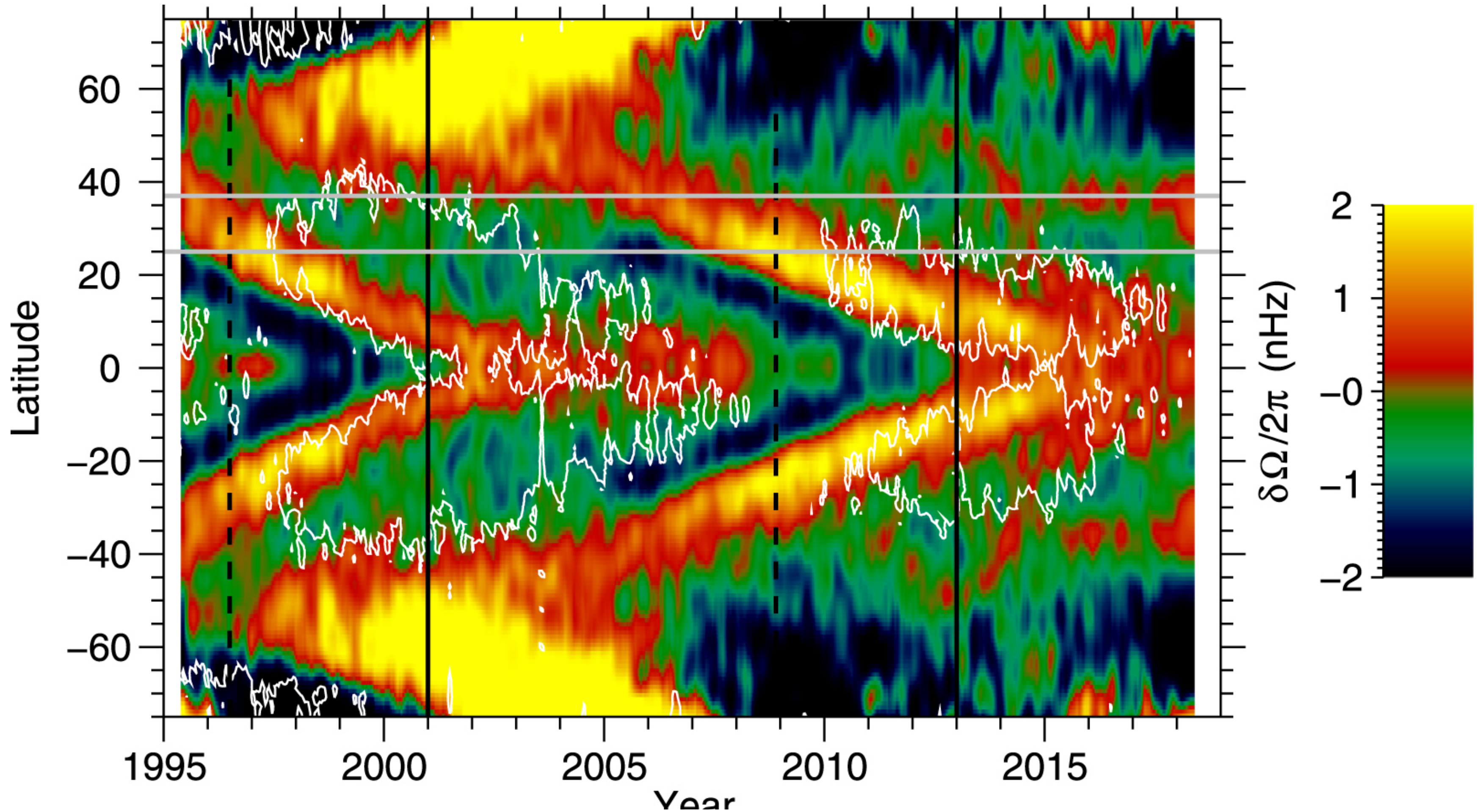




Antia & Basu 2013

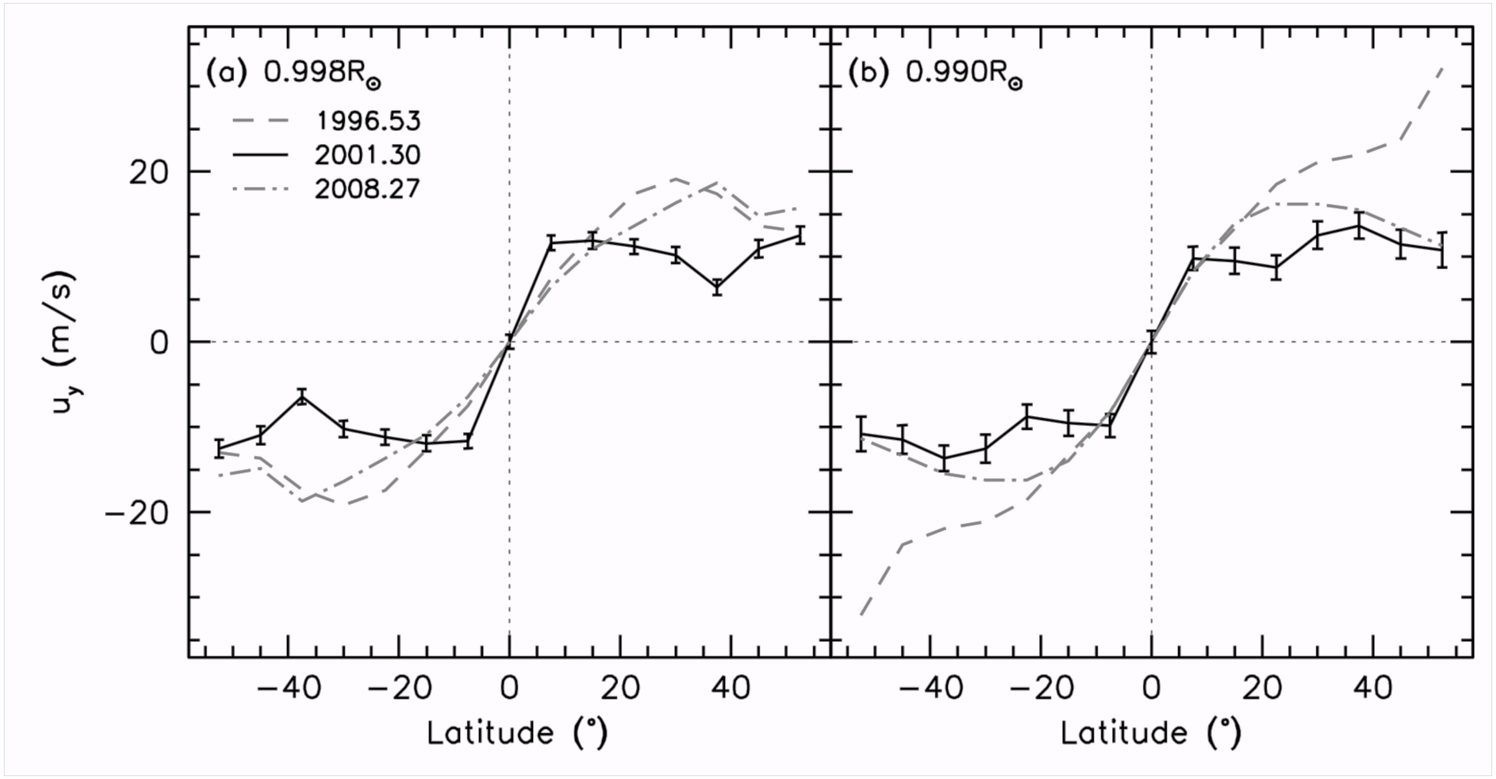


Antia & Basu 2008



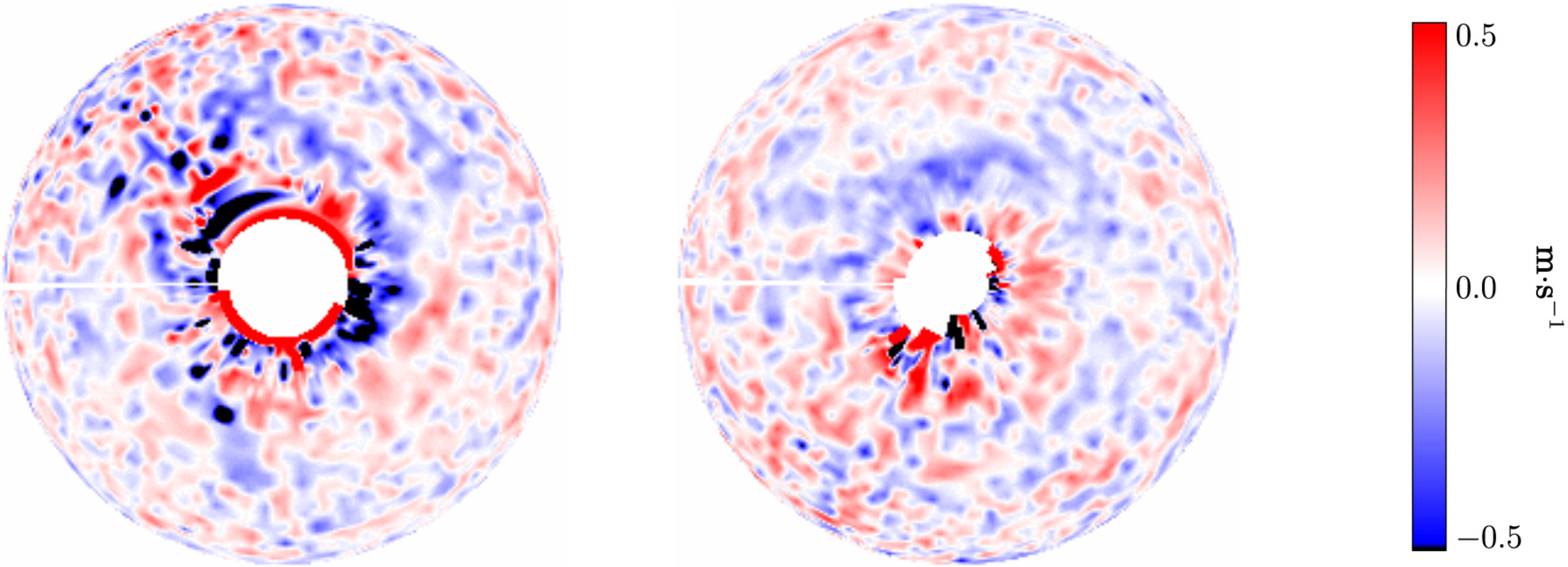
Howe et al. 2018

Meridional flows change too

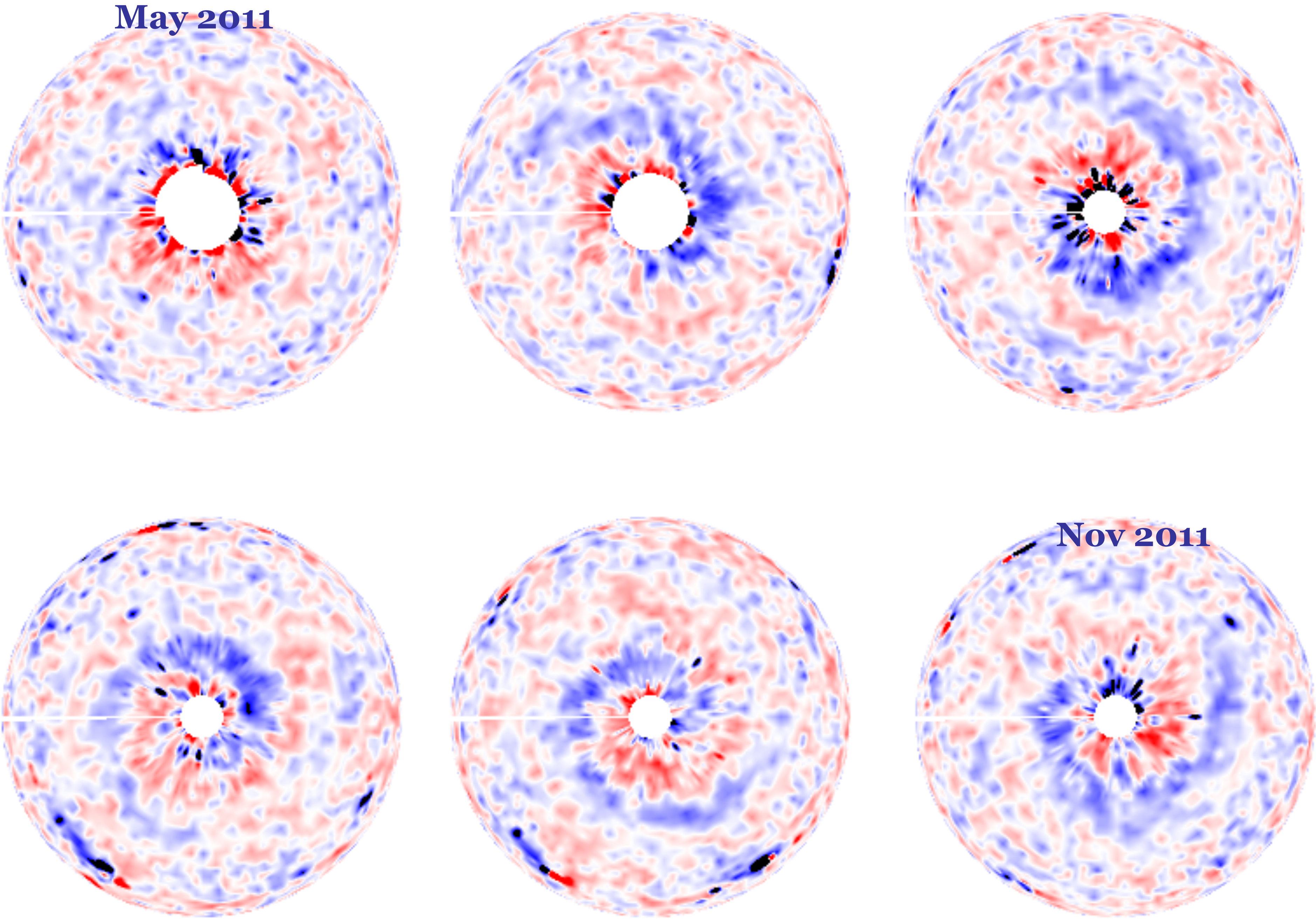


Basu & Antia
2010

And the swirls near the poles



From Bogart, Baldner & Basu 2015, 2018


May 2011
Nov 2011

What next?

- There are a number of unanswered questions when it comes to helioseismic analysis of the Sun:
  - Where do the meridional flows close?
  - What happens at the poles
  - What came first, the zonal flows of the sunspot butterfly diagram?