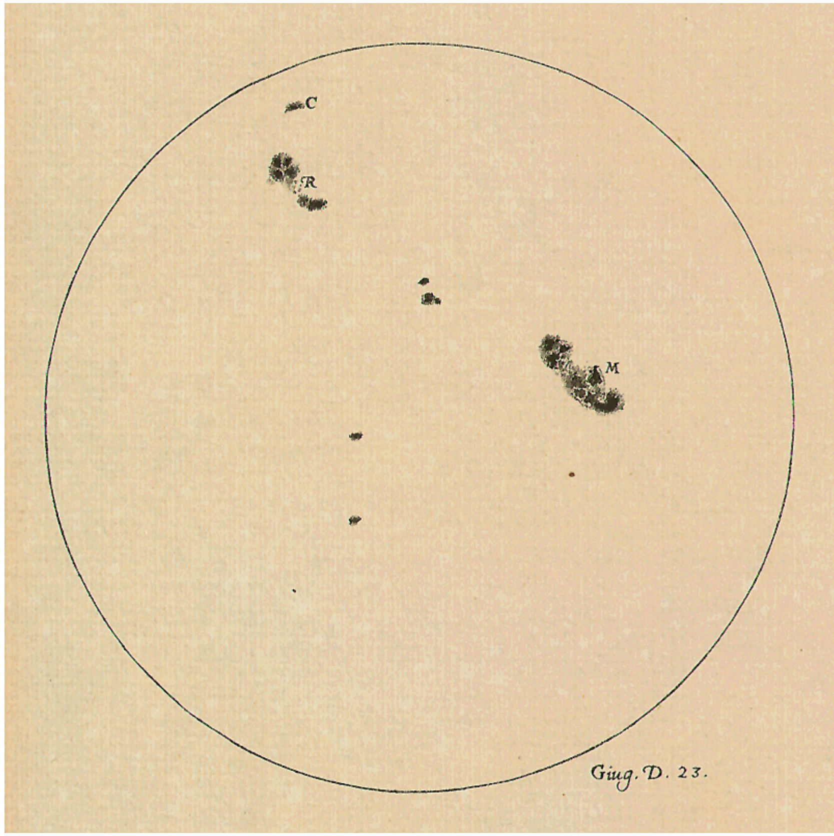
The Dynamo Mechanism in the Sun and Magnetic Helicity

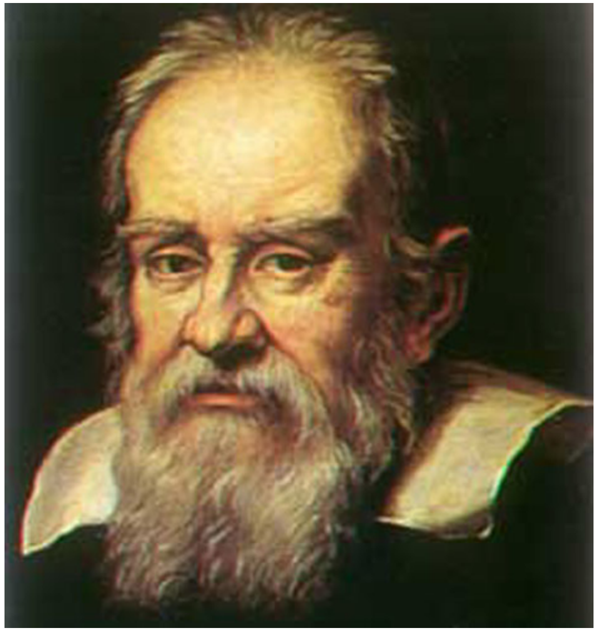
Kirill Kuzanyan[1,2)]

[1)]IZMIRAN, Moscow, Russia

[2)] Visiting Professor at National Astronomical Observatories, Chinese Academy of Sciences, Beijing,China

Galileo's Sunspot Drawings



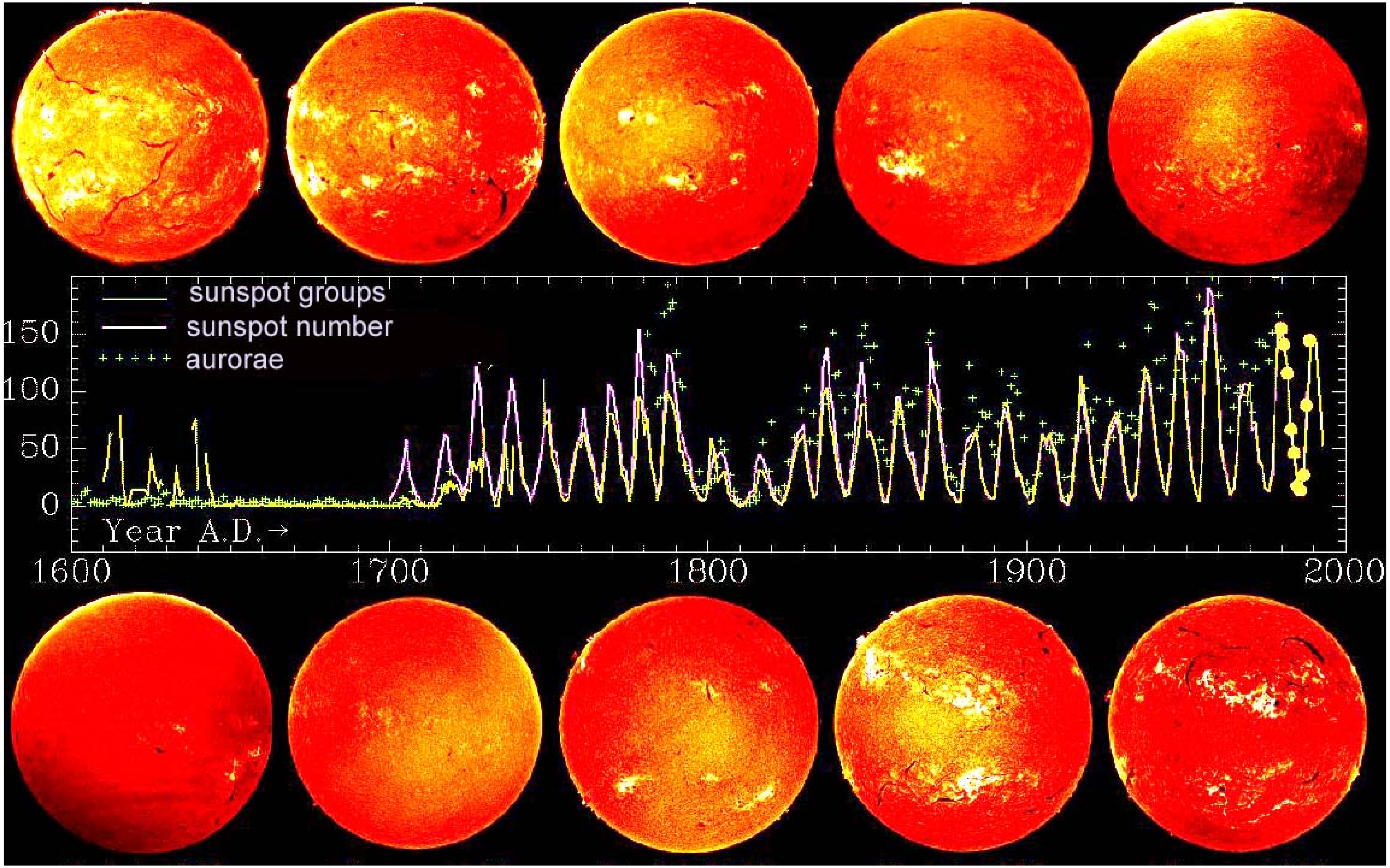


(*Galileo Galilei*)

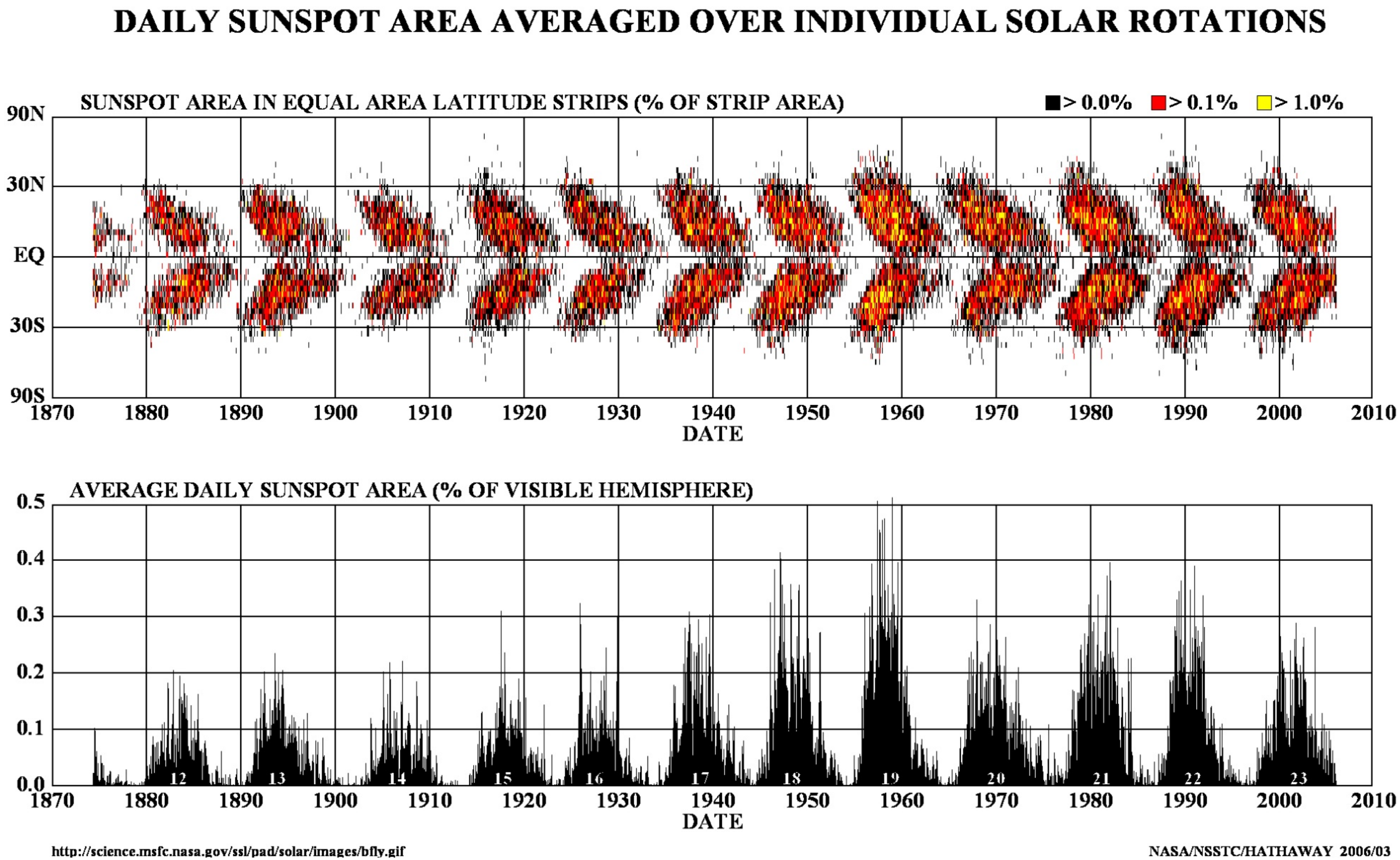
A series of sunspot observations during the summer 1612

published in *Istoria e Dimostrazioni Intorno Alle Macchie Solari e Loro Accidenti Rome* (History and Demonstrations; On Sunspots and their Properties, 1613)

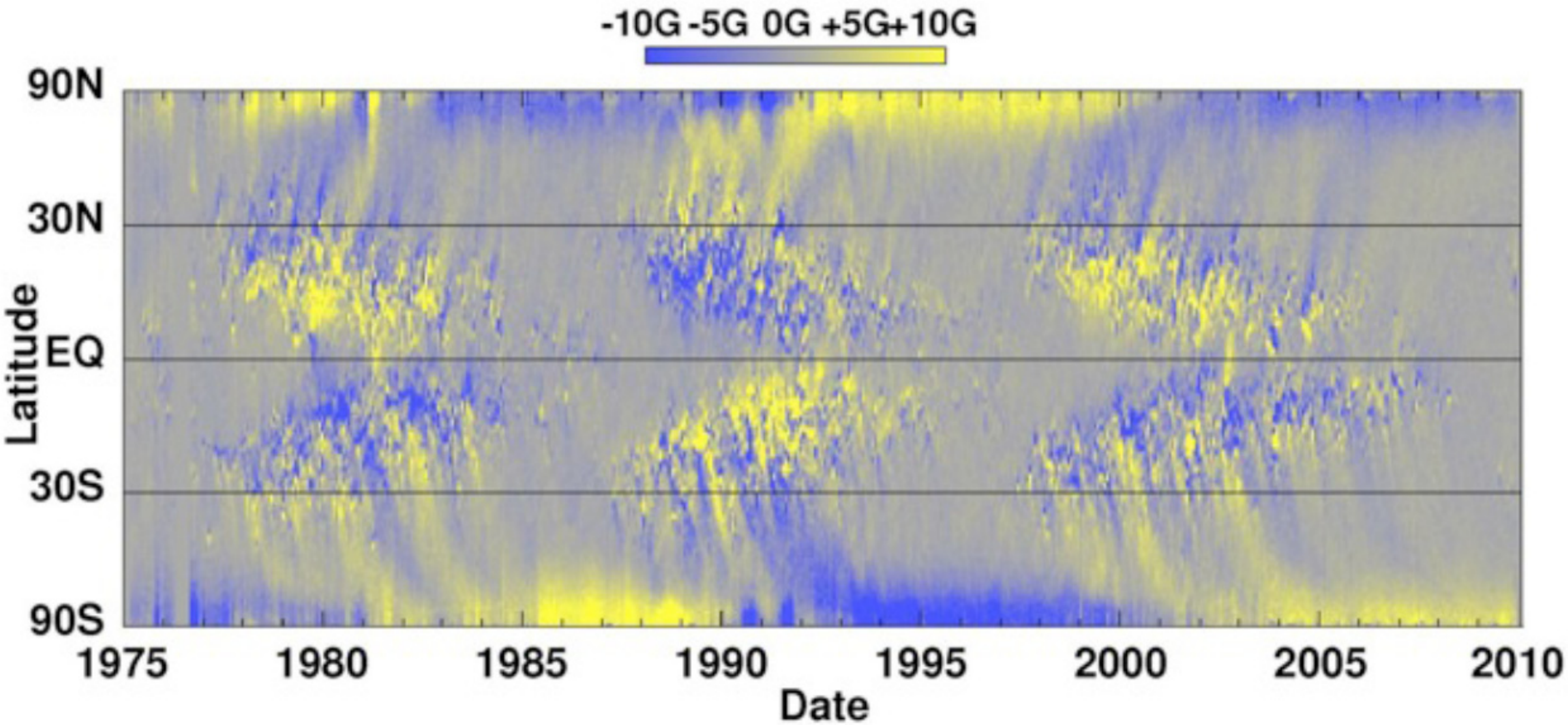
SOLAR ACTIVITY



The Butterfly Diagram

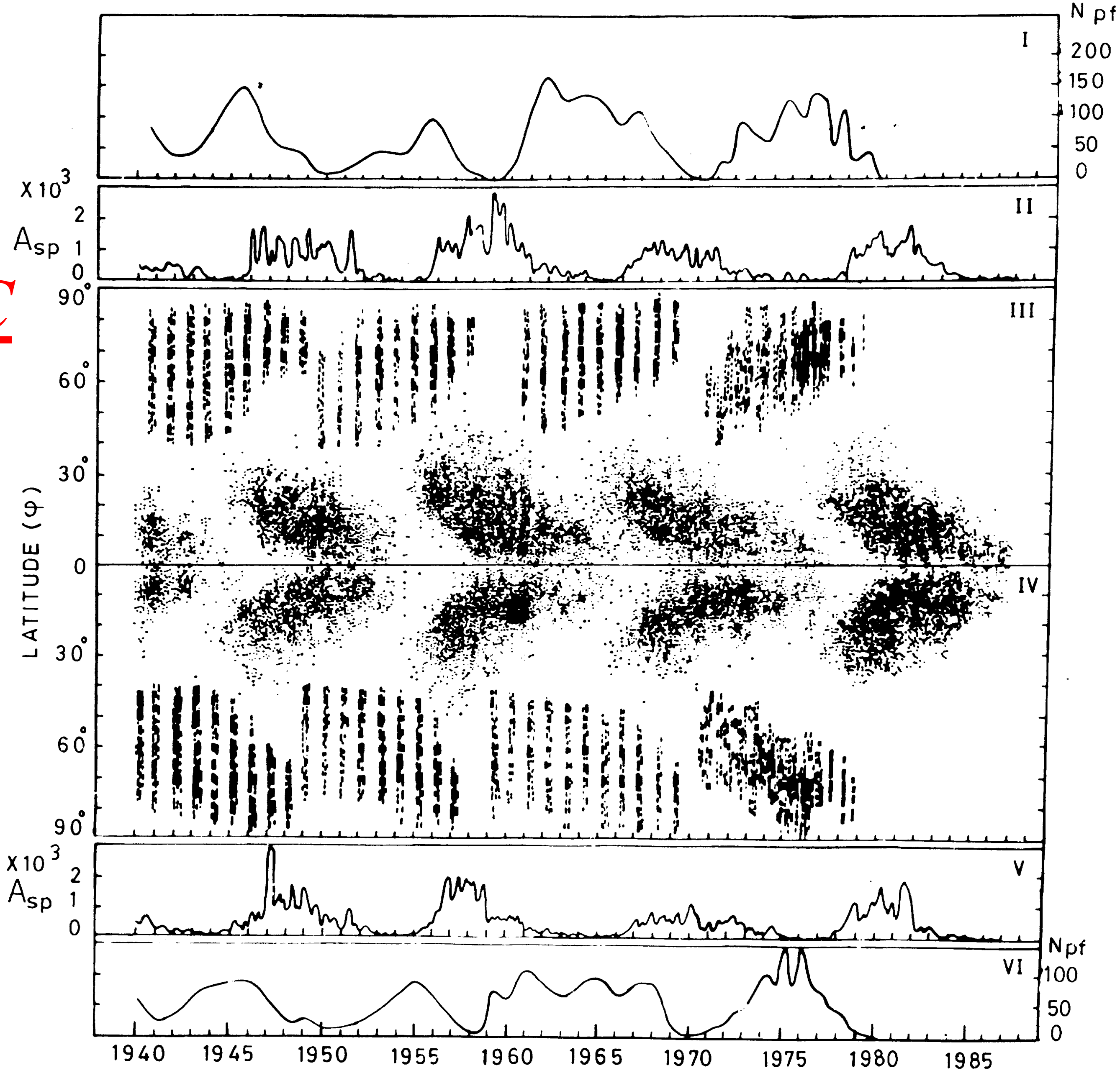


Large-Scale fields



(Image credit: NASA/MSFC/ David Hathaway)

POLAR MAGNETIC FIELDS



Makarov, Sivaraman, 1983,1989

PLAN of the talk

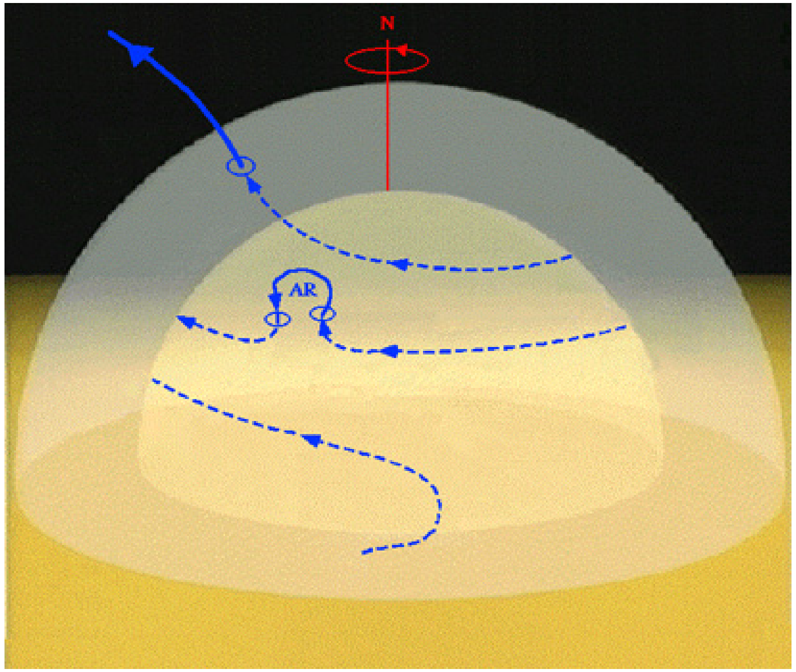
1. Solar Cyclic Activity: Large-Scale Magnetic Fields.
2. Stretch+Twist+Fold = DYNAMO
3. Parker Model for Dynamo Wave
4. MHD Equations: 3D numerical approach
5. Mean-Field Dynamo: notion of scale separation
6. The alpha-effect: 1D and 2D models
7. Asymptotic solution of Dynamo wave problem
8. Unknown: alpha[helicity], merid.circ.
9. Total Magnetic Helicity conservation law
10. Role of Magnetic Helicity in self-consistent dynamo
11. Anisotropic nature of solar magneto-convection: helicity and alpha-effect, overall magnetic field regeneration
12. Flux Transport Dynamo models: convenient simplification
13. Poloidal magnetic field: seed of the cycle
14. Double (multiple-) cell meridional circulation: results of helioseismology and mean field modelling
15. Formation of sunspots: NEMPI mechanism

17. Observational interpretations and Future prospects

Physics of the solar cycle

- **Solar dynamo theory**

Regeneration of magnetic fields due to rotation and turbulent convection



- periodic in time
- travelling wave

Parker 1955 dynamo wave

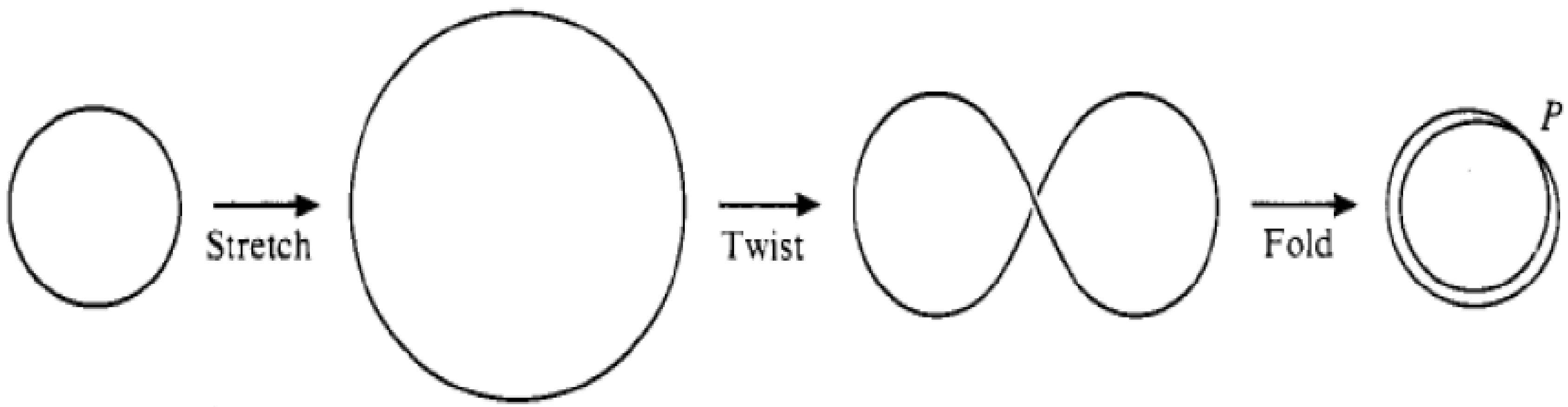
Babcock & Leighton 1961-69

Krause & Rädler 1980 mean-field model

Stretch-Twist-Fold Dynamo

Ya.B. Zel'dovich 1970s

(from H.K.Moffat, 1978 etc.)

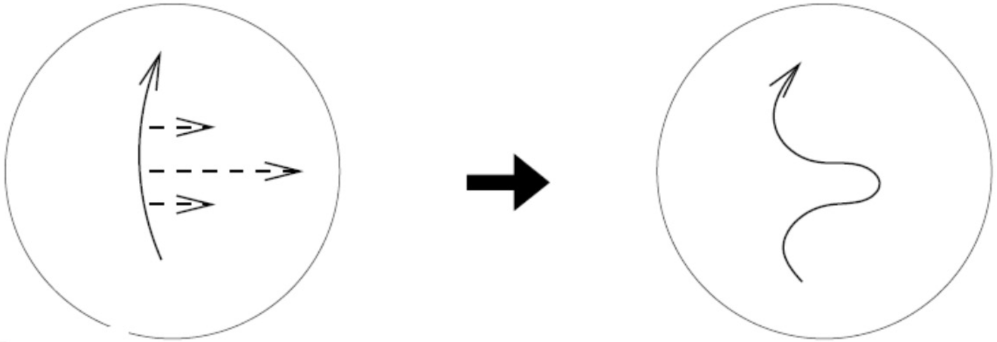


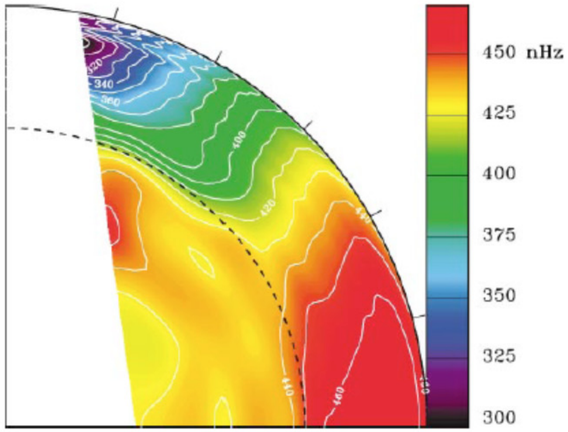
Differential rotation

At large Rm, magnetic fields are advected with the flow

This implies that any shearing motions tend to deform magnetic fields in the direction of the flow

Illustrative sketch:





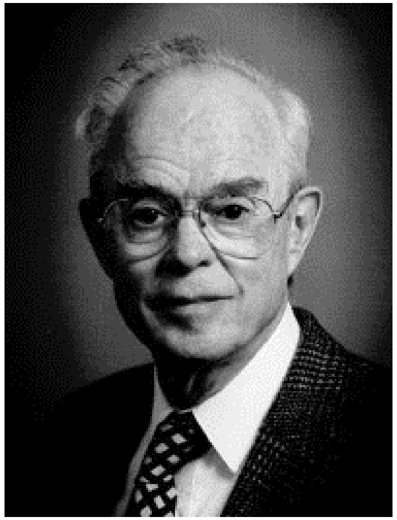
In the case of the Sun (left):

Differential rotation will tend to shear out magnetic fields in the azimuthal direction

Migratory Dynamo wave model

Magnetic field generation
(Parker Dynamo)

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial \theta^2} & \alpha(\theta) \\ -D\cos\theta\frac{\partial}{\partial \theta} & \frac{\partial^2}{\partial \theta^2} \end{bmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$



E.N. Parker (1955)

Basic equations of solar magnetism

- Solar convection zone governed by equations of compressible MHD

$$P = R\rho T \text{ (Perfect Gas)} \quad \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \text{(Continuity)}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \text{ (Induction)}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) = -\nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \nabla P + \rho \mathbf{g} + \nabla \cdot \tau + \mathcal{F}_{other} \text{ (Momentum)}$$

$$(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla)\left(\frac{P}{\rho^{\gamma}}\right) = \text{Source and Loss terms (Energy)}$$

Solar Parameters (Ossendrijver 2003)

	BASE OF CZ	**PHOTOSPHERE**
$Ra \equiv g\Delta\nabla d^4 / \nu\chi H_P$	10^{20}	10^{16}
$\mathrm{Re} \equiv UL/\nu$	10^{13}	10^{12}
$Rm \equiv UL/\eta$	10^{10}	10^{6}
$\mathrm{Pr} = \nu/\chi$	10^{-7}	10^{-7}
$\beta = 2\mu_0 p / B^2$	10^{5}	1
$Pm = \nu/\eta$	10^{-3}	10^{-6}
$M = U/c_s$	10^{-4}	1
$Ro = U/2\Omega L$	0.1-1	10^{-3}-0.4

Direct numerical simulation

unfortunately, no coherent magnetic structure obtained!

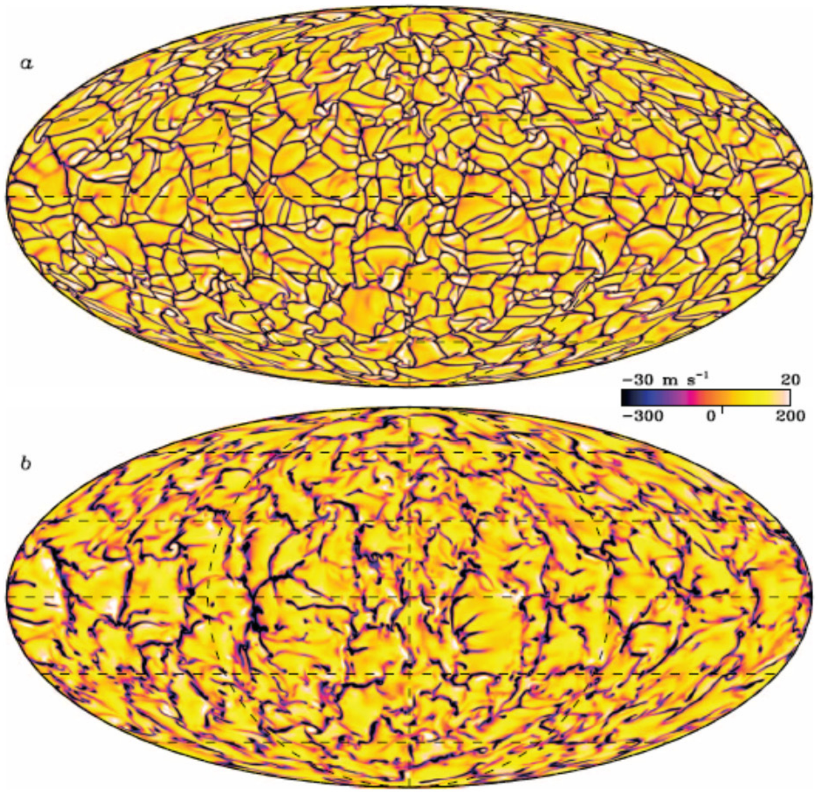
Theoretical models (large-scale numerical simulations):

The most successful current models are anelastic simulations of the convection zone (no radiative interior, no tachocline)

"ASH code": Anelastic Spherical Harmonics

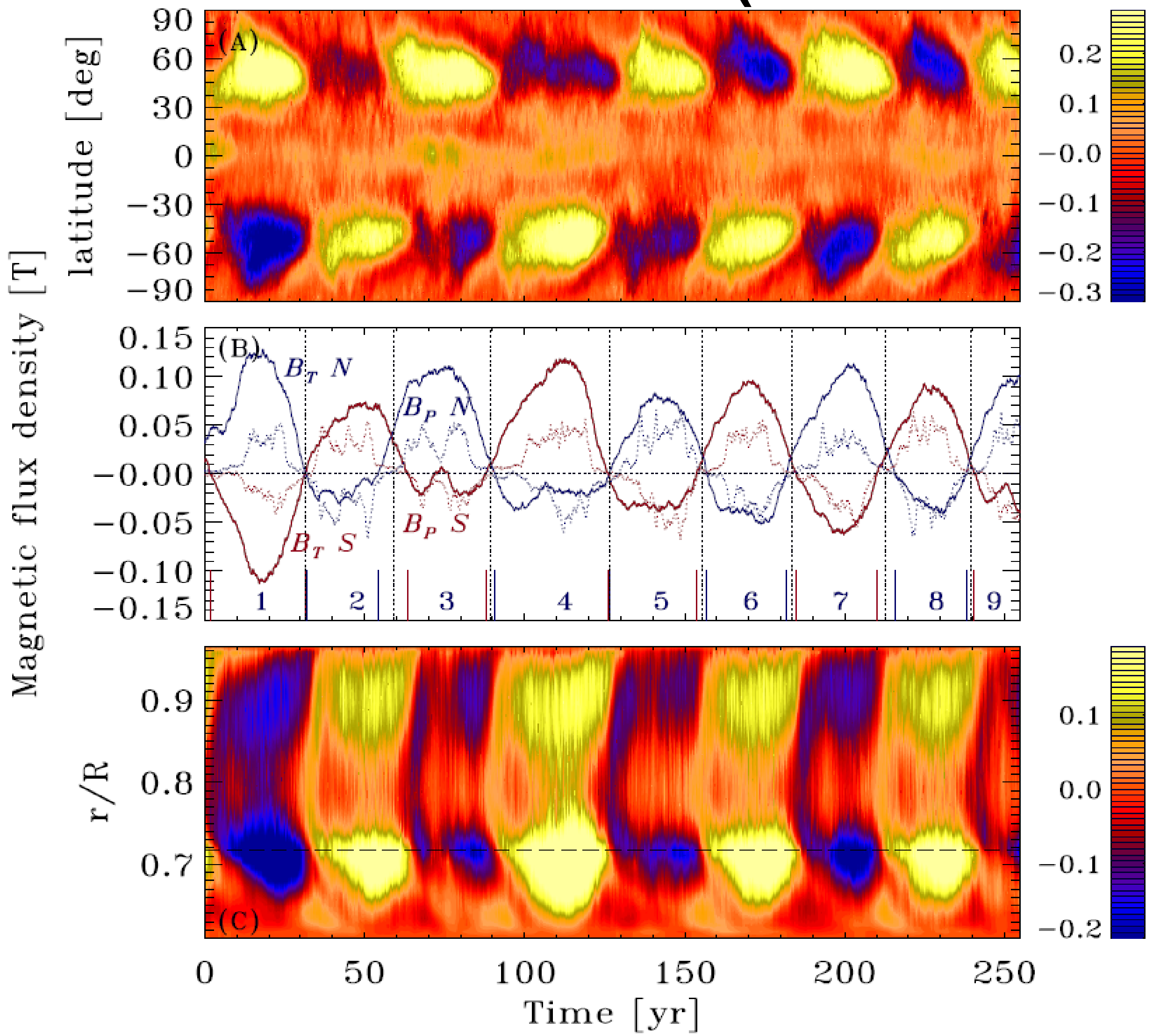
Right: (Miesch et al. 2008)

Projections of the convective radial velocity in a spherical shell at r=0.98 (top) and r=0.95 (bottom)



Resolution: 5×10^8grid points

3D GLOBAL LARGE-EDDY SIMULATIONS (*Ghizaru et al. 2010*)



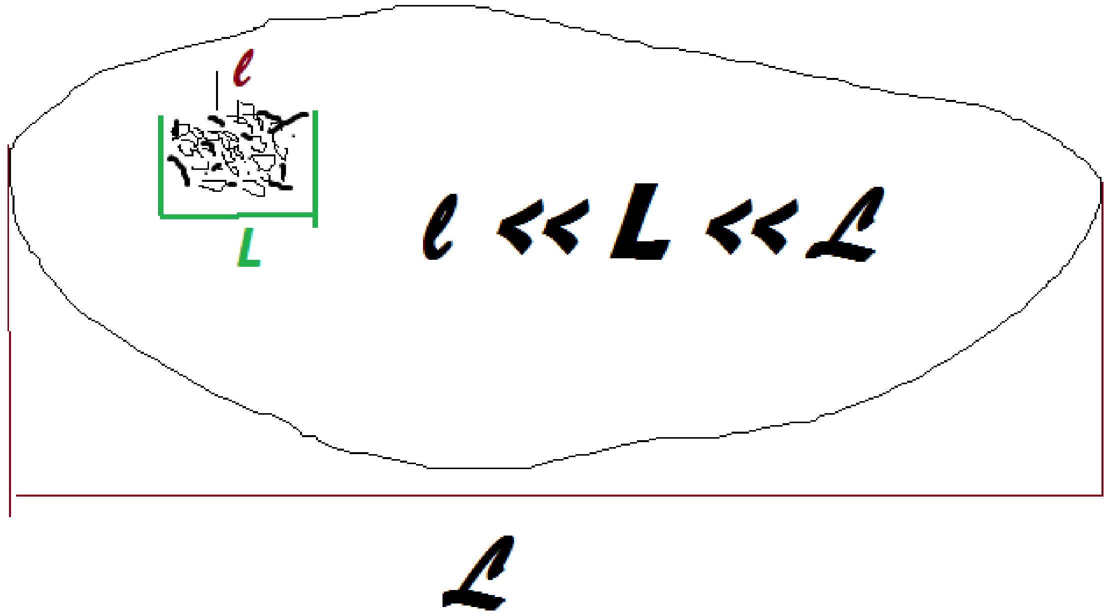
Look like magnetic cycles but

<u>Standing rather than travelling wave;</u>

nobody knows why!?!

towards Mean-Field theory

- *notion of sclale separation!*



Founders of mean-field dynamo theory

Max Steenbeck, Fritz Krause, Karl-Heinz Raedler
Potsdam, Germany, 1966 - ….

Scale separation: "Mean" scales $<\ldots>_{l,t}$

- ***Latitude***

  sunspot << "mean" << Convective Zone

  (granula) (or Solar Radius)

- ***time***

  1 day << "mean time" << Solar Cycle

“Mean-field” ***scales***

- Smaller than entire astrophysical body (the Sun)

  10^7-10^9 cm << ***L*** << 10^{11} cm

  1-10 days << ***T*** << 10^4 days

- Larger than fluctuation level (granulae)

Turbulent Diffusion and Scales

- Spatial and time scales are linked by turbulent diffusivity (eta) = L^2/ (tau)

For the Sun (eta) ~ 10^{12}-10^{14} cm^2/s

check it on a range of scales and times

"Mean" scales are less than entire scales of the object but bid enough, compared with "the background", and so observable

Mathematically: **averaging over the ensemble of turbulent pulsations**

Mean-Field dynamo theory

Parker's model is based on physical intuition. It is possible to derive the α-effect in a more rigorous fashion (see e.g. Moffatt 1978)

Recall:
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

Decompose the magnetic field and the velocity field into mean and fluctuating parts (angled brackets denote an average):

$$\mathbf{B} = \mathbf{B}_0 + \mathbf{b} \qquad \mathbf{U} = \mathbf{U}_0 + \mathbf{u}$$

$$\mathbf{B}_0 = \langle \mathbf{B} \rangle \qquad \mathbf{U}_0 = \langle \mathbf{U} \rangle$$

$$\langle \mathbf{b} \rangle = \mathbf{0} \qquad \langle \mathbf{u} \rangle = \mathbf{0}$$

Turbulent Electro-Motive Force

Substitute these into the induction equation and average to get:

$$\frac{\partial \mathbf{B}_0}{\partial t} = \nabla \times \langle \mathbf{u} \times \mathbf{b} \rangle + \nabla \times (\mathbf{U}_0 \times \mathbf{B}_0) + \eta \nabla^2 \mathbf{B}_0$$

New Term

If the velocity is unaffected by the magnetic field, then it can be shown that this new term is linearly related to the mean field, i.e.

$$\langle \mathbf{u} \times \mathbf{b} \rangle_i = \alpha_{ij} B_{0j} - \beta_{ijk} \frac{\partial B_{0k}}{\partial x_j} + \text{higher order terms}$$

MEAN FIELD MAGNETO-HYDRODYNAMICS

GOVERNING EQUATION for **B**
(M. Steenbeck, F. Krause и K.-H. Rädler, 1966):

$$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{curl}\, \alpha \mathbf{B} + \mathbf{curl}\, [\mathbf{V} \times \mathbf{B}] - \mathbf{curl}\, \beta \mathbf{curl} \mathbf{B},$$

$\mathbf{V}$ = Differential Rotation (Ω–effect),

$\alpha \sim -\frac{\tau}{3} < \mathbf{u}' \cdot \mathbf{curl}\, \mathbf{u}' >$ = α–effect

mean (flow) helicity,

β turbulent diffusivity.

Axial symmetry

=> Decomposition into toroidal and poloidal parts:

$$\mathbf{B} = \mathbf{B}_p + \mathbf{B}_t,$$

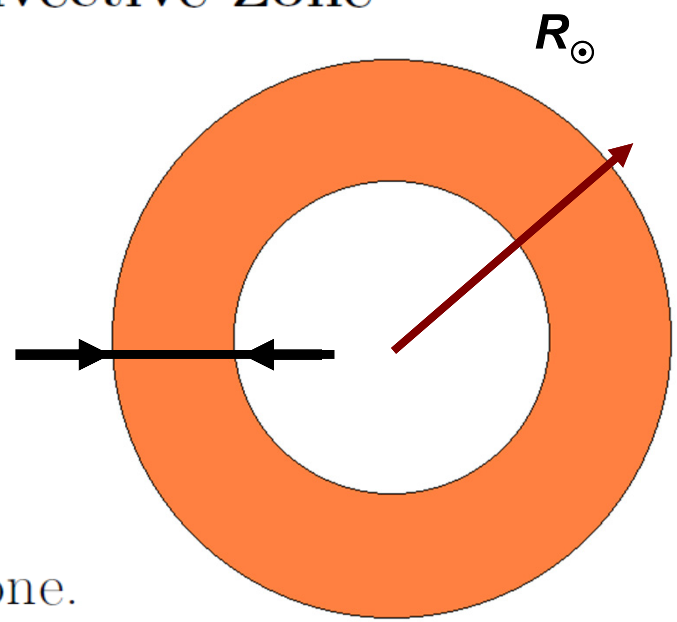
where

$$\mathbf{B}_p = \mathbf{curl}(0, 0, A(r, \theta, t)),$$

$$\mathbf{B}_t = (0, 0, B(r, \theta, t)).$$

$$\mathbf{B}_t \xrightarrow{\alpha} \mathbf{B}_p \qquad \mathbf{B}_p \xrightarrow[\alpha]{\Omega} \mathbf{B}_t$$

THIN SHELL – Solar Convective Zone



$$\lambda = \frac{h}{R} \ll 1\,,$$

h thickness of the Convective Zone.

1D approach main assumption

(thin shell – short waves –
high turbulence – strong generation source):

$$|D|^{-1/3} \ll \lambda \ll 1$$

D = **dynamo number**
= intensity of generation.

i.e. high magnetic Reynolds number

Parker's dynamo equations (1955):

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial \theta^2} & \alpha(\theta) \\ -D\cos\theta \frac{\partial}{\partial \theta} & \frac{\partial^2}{\partial \theta^2} \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

note: Large dimensionless parameter $|D| \gg 1$

not a self-adjoint operator!

Quantum theory analogues

$$\frac{\partial}{\partial t}\psi = \widehat{\mathcal{H}}\,\psi$$

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial\theta^2} & \alpha(\theta) \\ -D\cos\theta\frac{\partial}{\partial\theta} & \frac{\partial^2}{\partial\theta^2} \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

- U

potential $\widehat{\alpha} = \alpha(\theta)\cos\theta$

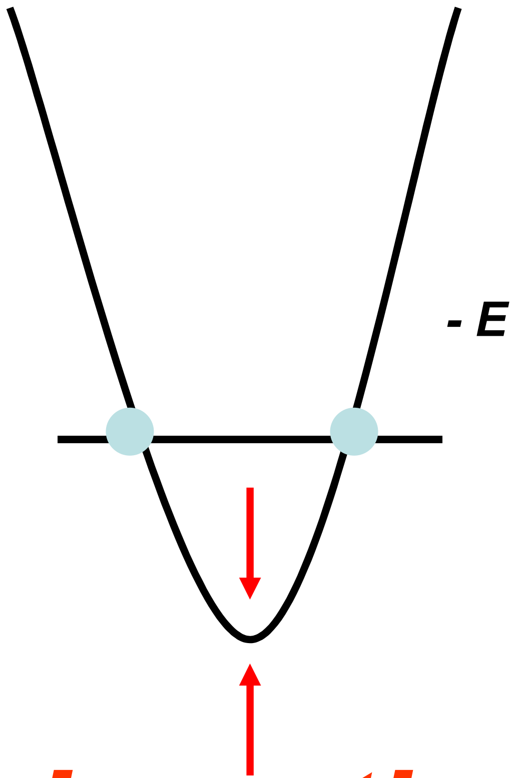
- E

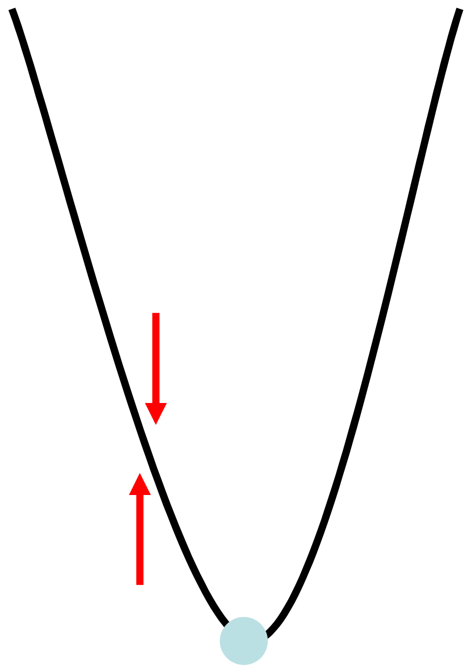
Energy levels γ

$$\psi \sim \exp\left(iS/\hbar\right)$$

$$\exp\left(\frac{iS}{|D|^{-1/3}} + \gamma t\right)$$

Turning points





Location of the solution

Semi-classical approximation

$\psi \sim \exp\left(iS/\hbar\right)$ in quantum theory

is usually applicable for the base level of energy (leading mode 0) as well as higher order modes.

There comes only one turning point !
(see V. Maslov)

now short waves!

(and the maximum of the solution is not localized at the turning point!)

Asymptotic solution

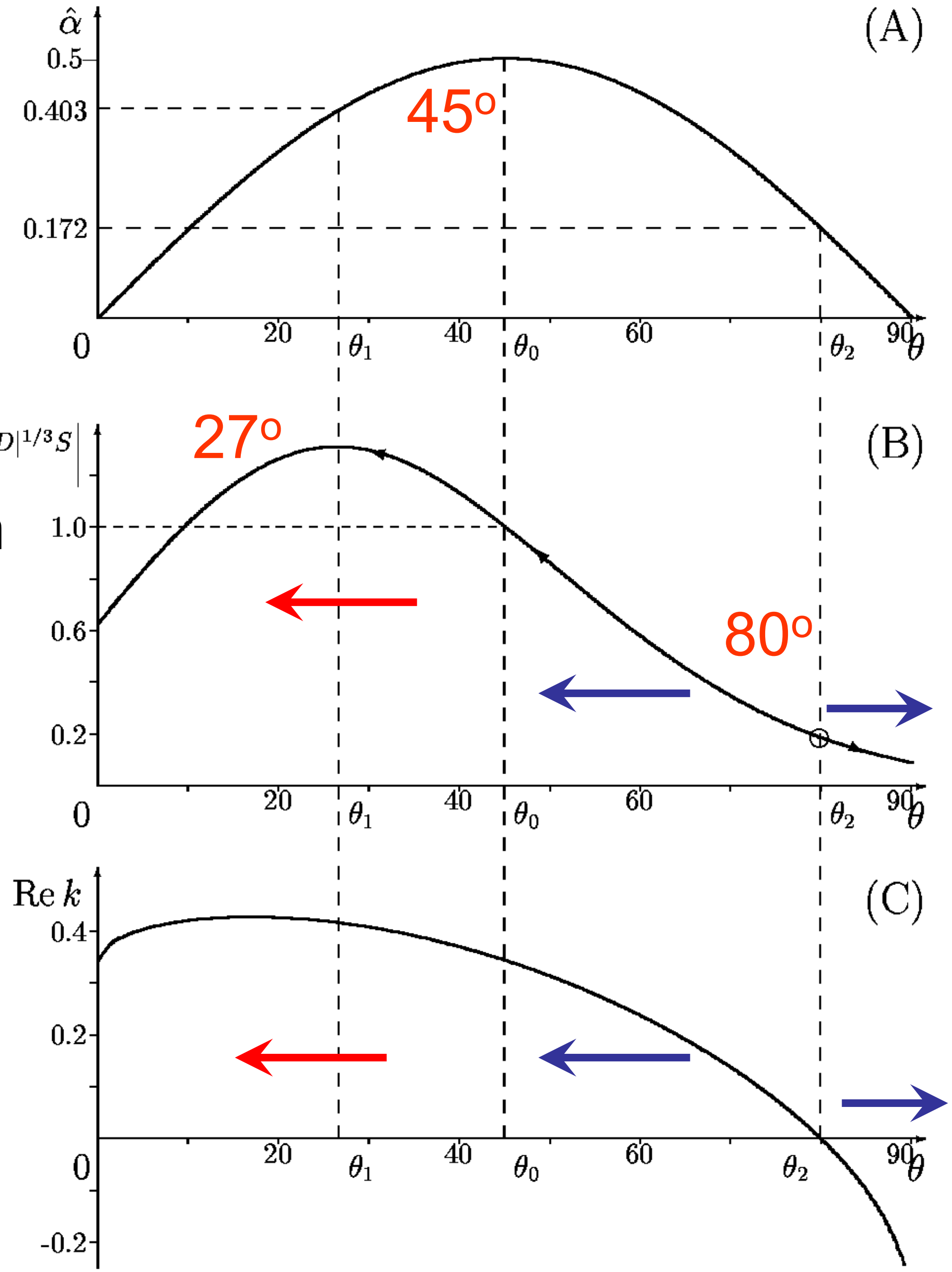
Generation Source (potential) α^

Envelope of the solution

$$\exp\left\{ i \left|D\right|^{1/3} \int k(\theta)\, d\theta \right\}$$

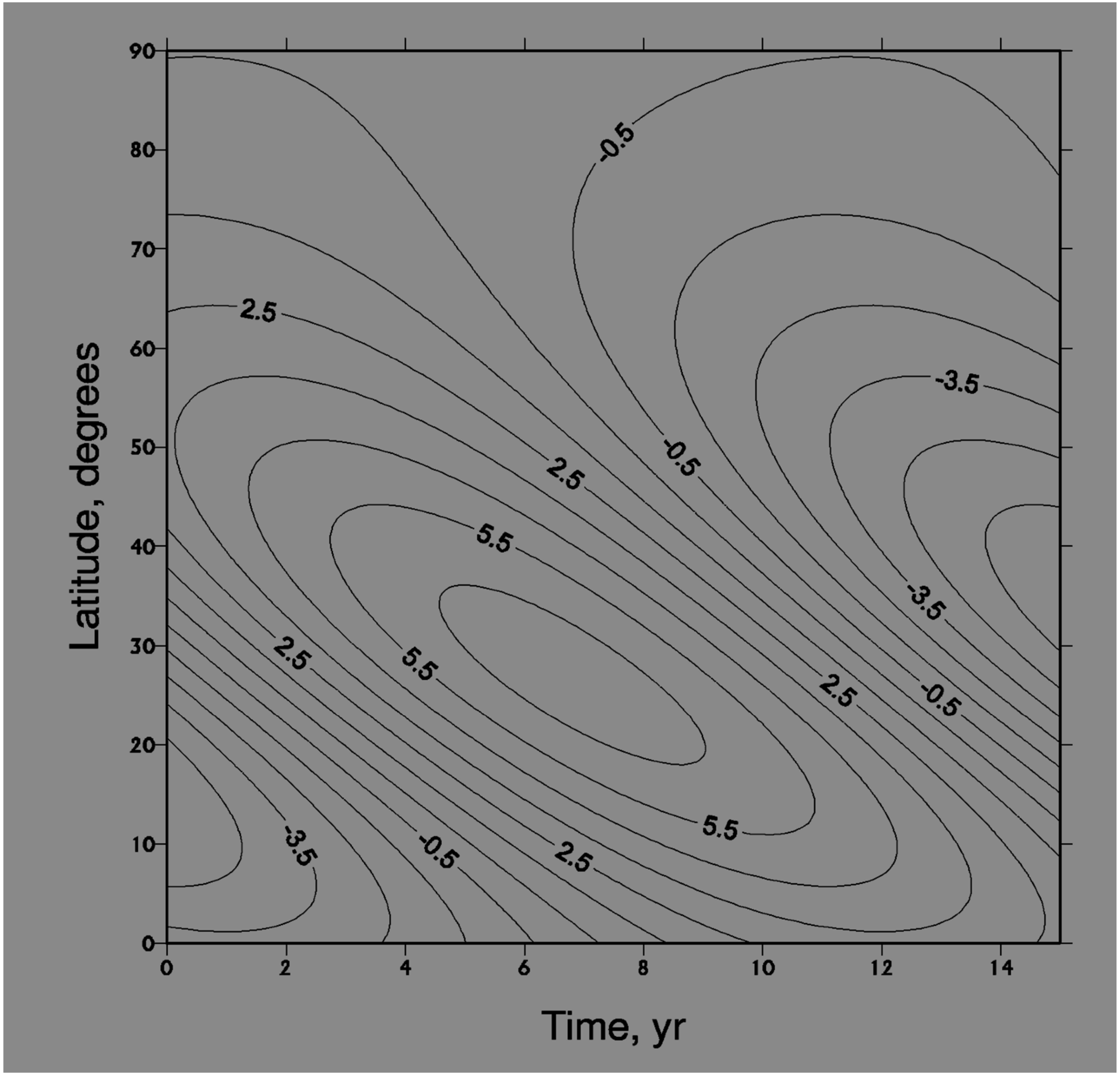
Kuzanyan, Sokoloff (1995)

Wave number (real part of)



Butterfly diagram

(Kuzanyan and Sokoloff 1997)

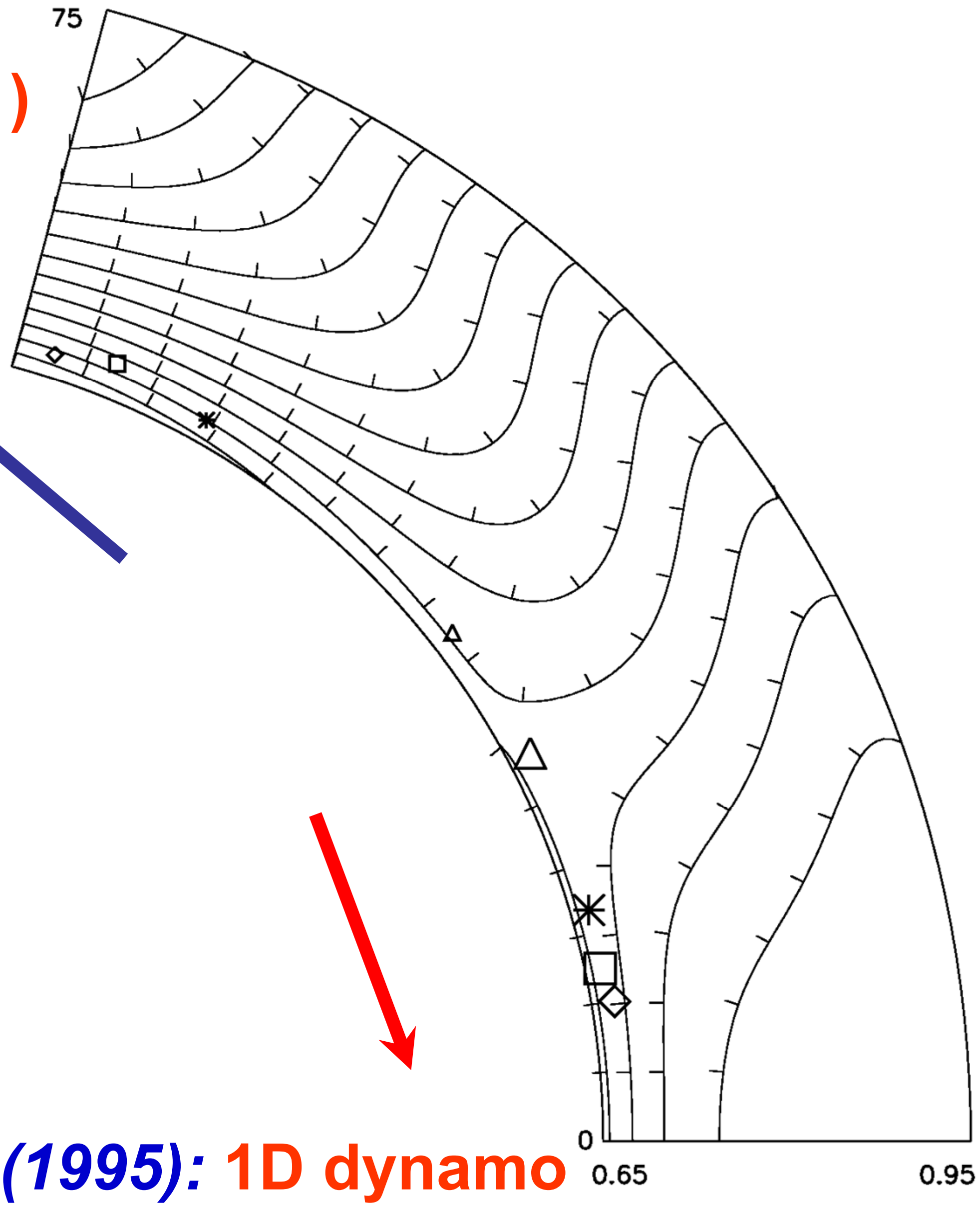


Belvedere, Kuzanyan, Sokoloff (2000): **analytic 2D dynamo with the two waves (equatorward and poleward)**

angular rotation

$$\Omega(r,\theta) = \sum_{j=0}^{2} \cos 2j\theta \sum_{i=0}^{4} c_{i,j} r^i,$$

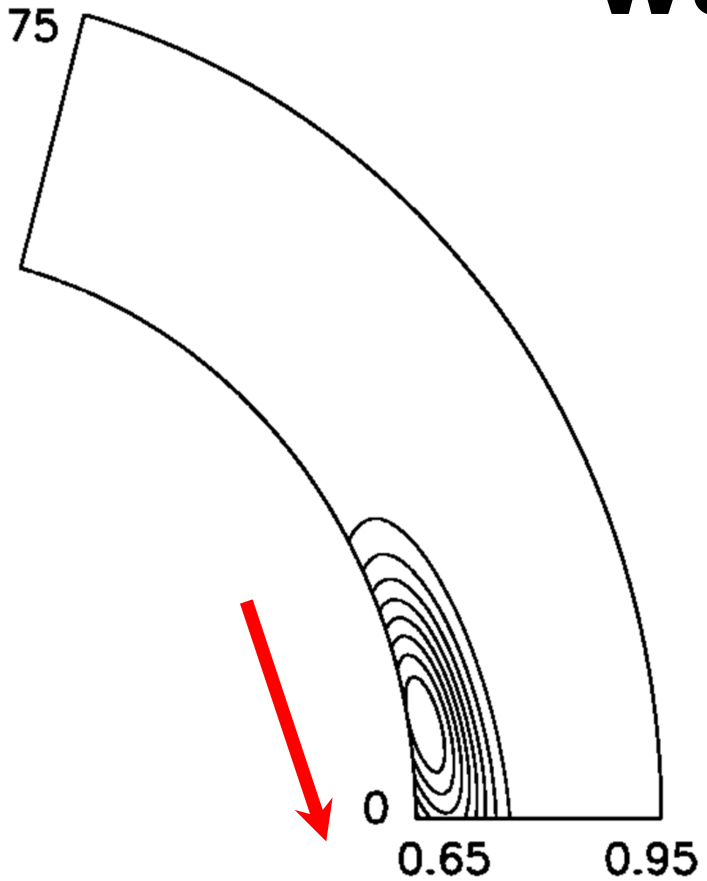
$$(0 \leqslant \theta \leqslant 75^\circ, \quad 0.65 \leqslant \frac{r}{R_\odot} \leqslant 0.95)).9$$

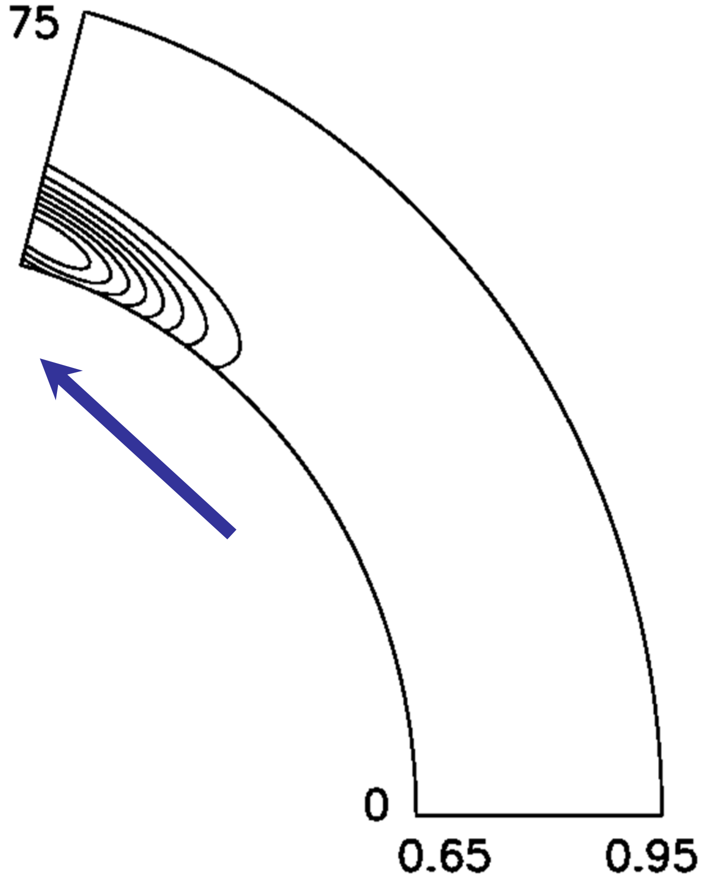


- ✳ maximum source
- △ turning of the wave
- ◇ maximum 1D solution
- □ maximum 2D solution

after *Kuzanyan and Sokoloff (1995):* 1D dynamo

Two waves





equatorward poleward

related to 1D solution

Two waves: sunspots and polar faculae

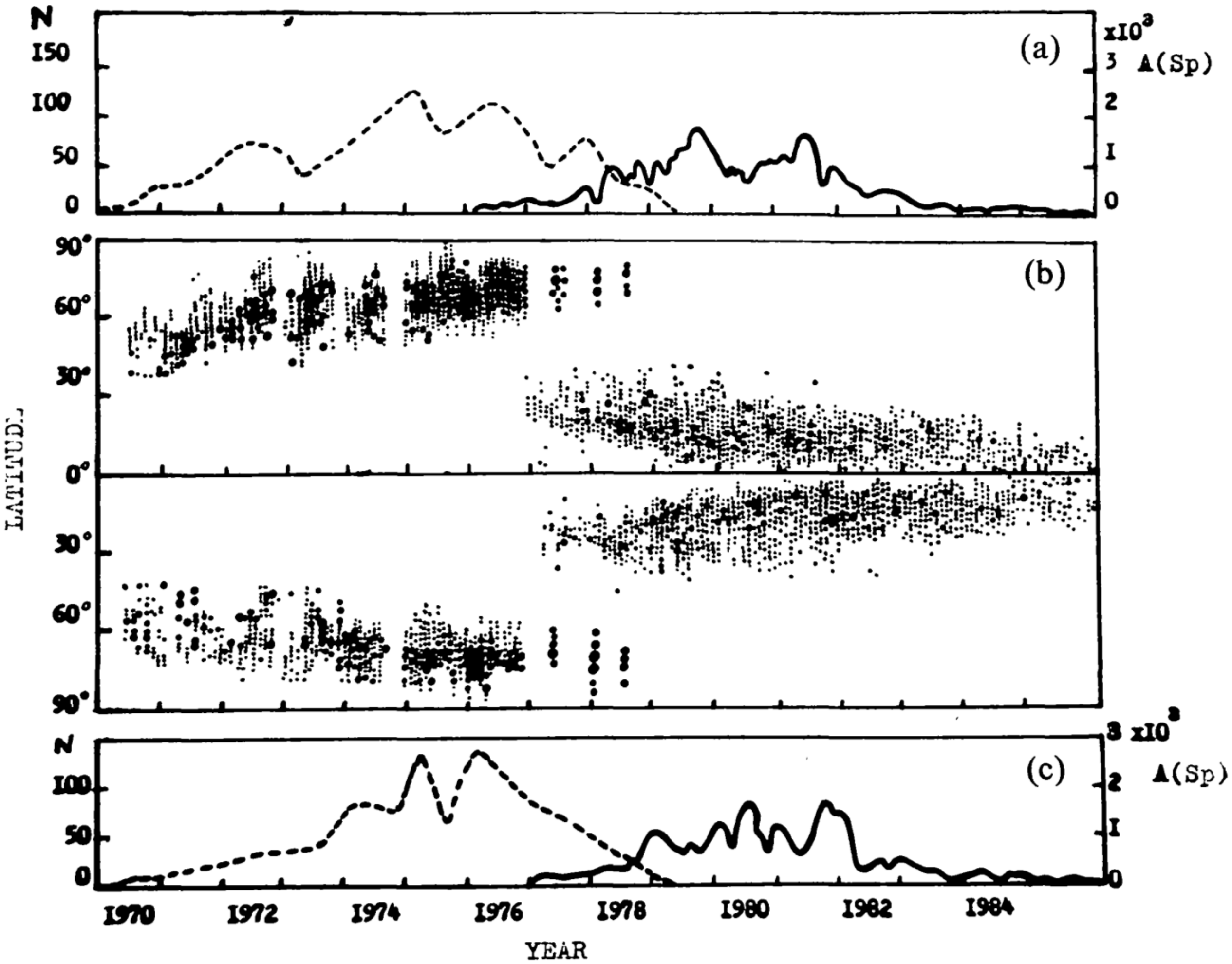
Two dynamo waves

- **Low latitudes (max at ~ 16°): sunspot butterfly diagram**
- **High latitudes (max at ~61°): polar faculae**

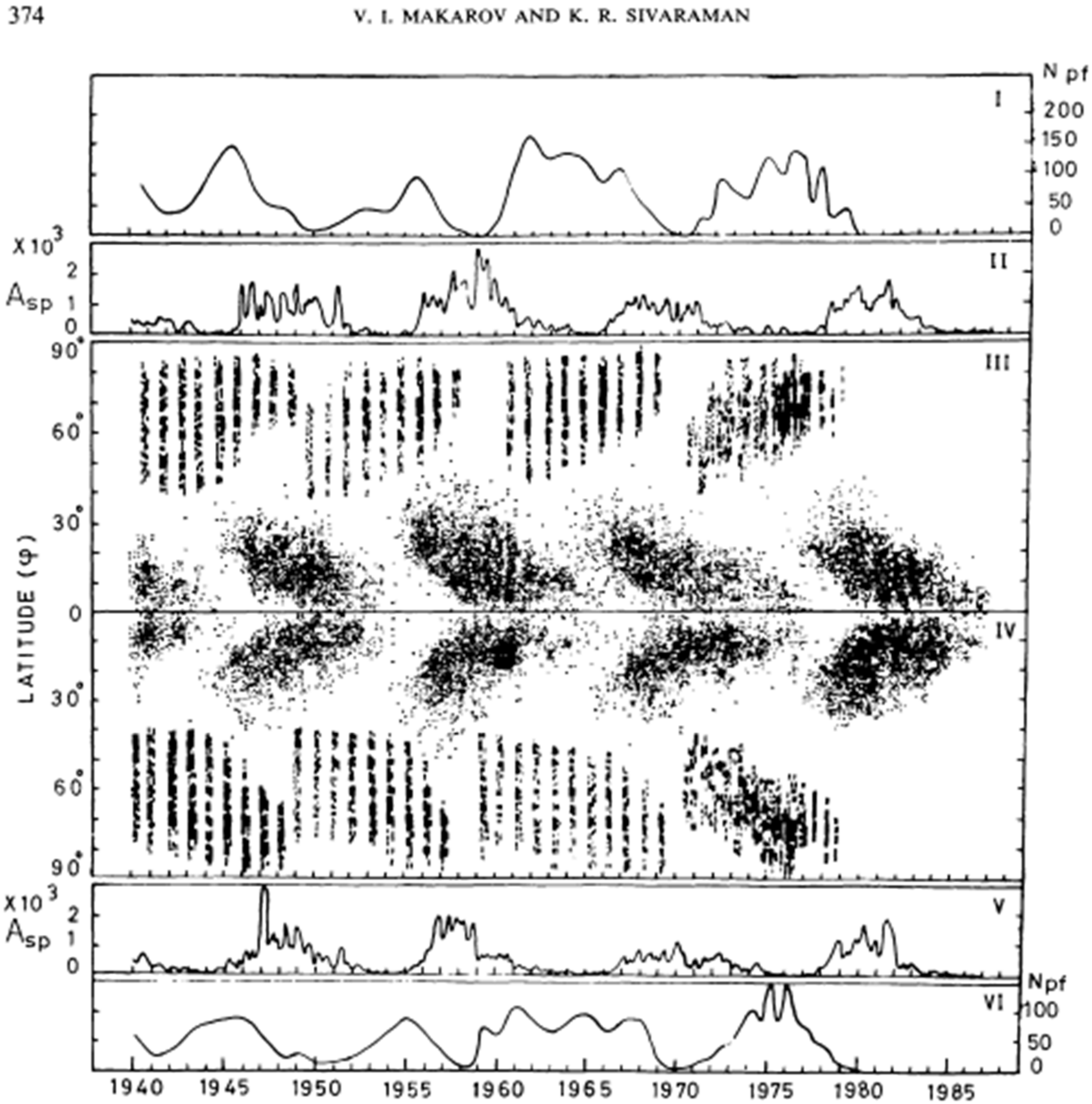
Why polar…?

- **The polar magnetic fields are systematically observed for 60+ years by means of polar faculae and bright points (from Makarov and Sivaraman, 1981 and afterwards).**
- **The polar branch of the dynamo wave have been theoretically investigated**

Makarov et al. 1987: polar faculae versus sunspot cycle



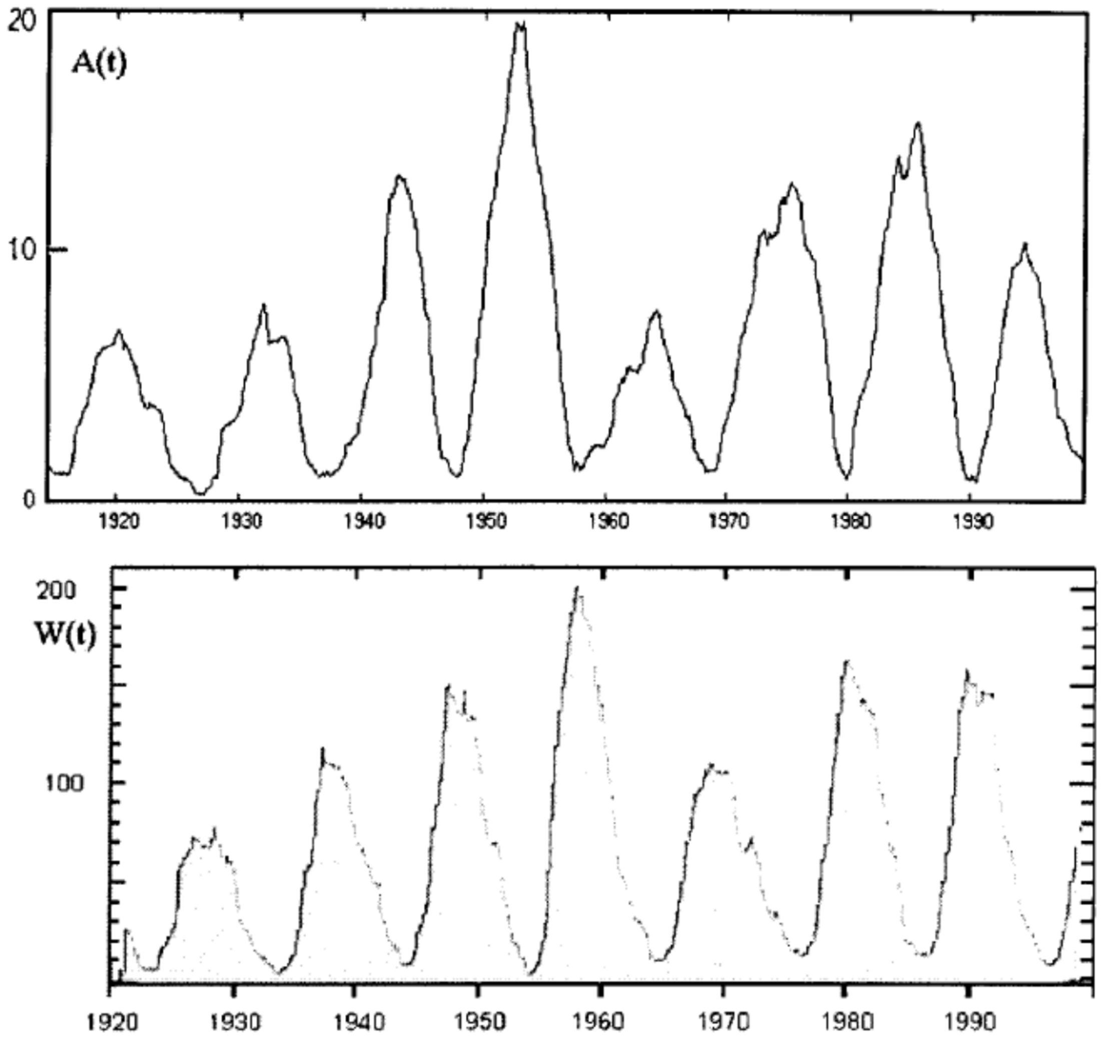
1987-88: some theoretical modelling with A.A. Ruzmaikin



Makarov & Sivaraman (Sol. Phys. 1989)

Observations of polar faculae: phase shift with the sunspots

Makarov et al. (Sol. Phys. 2001)



Large-Scale fields and Sunspot Cycle:

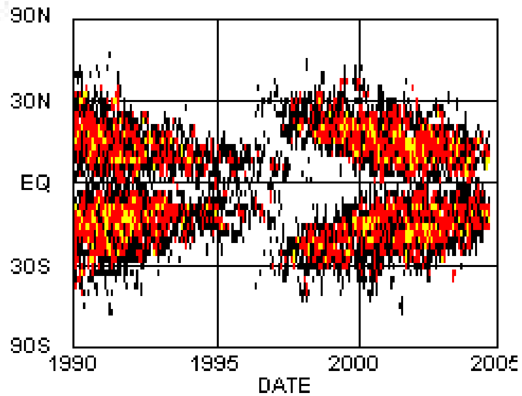
half-cycle time lag

Simple self-consistent dynamo models with evolution of helicity (dynamical nonlinearity)

Kleeorin, Kuzanyan, Moss, Sokoloff, Rogachevskii, Zhang, A&A, 2003; and a series of publications of the authors thereafter in 2005-2011

The role of helicity and the α-effect

1. EQUATORWARD Dynamo Wave propagation (i.e. Parker 1955: α-effect – regeneration of poloidal fields from toroidal ones)



North $\alpha \frac{\partial \Omega}{\partial r} < 0$

South $\alpha \frac{\partial \Omega}{\partial r} > 0$

$\Omega(r, \theta)$: results of Helioseismology: $\frac{\partial \Omega}{\partial r} > 0$

We cannot directly measure the α-effect in S.C.Z.!

“Residual” helicity α-effect

- Helicity balance near the saturation limit

“Residual” α-effect
(after Frisch, Pouquet et al, 1975)

$$\alpha \sim C_1\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle + C_2\langle \mathbf{v_c} \cdot \nabla \times \mathbf{v_c} \rangle$$

Both helicities are of the same order of magnitude and the same sign (*C_1* and *C_2* have opposite signs), for developed dynamo they nearly cancel each other.

Correlation of Helicities

high conductivity limit

$$\mathbf{v_c} \cdot \nabla \times \mathbf{v_c} \qquad \sim \text{correlation} \rightarrow \underline{\mathbf{b} \cdot \nabla \times \mathbf{b}}$$

where

$\mathbf{b}$ small scale magnetic field

$H_c = \mathbf{b} \cdot \nabla \times \mathbf{b}$ Current Helicity

Estimate of the α-effect

$$\alpha \sim -\langle \mathbf{b} \cdot \nabla \times \mathbf{b} \rangle \sim -\langle \mathbf{v_c} \cdot \nabla \times \mathbf{v_c} \rangle$$

(Keinigs 1983; Rädler and Seehafer 1990; Seehafer 1994 etc.)

The Role of Helicities in Dynamo

- ***Inviscid integrals***

  magnetic helicity

  $\boldsymbol{A}\cdot\boldsymbol{B}$ **(for turbulent motion, at all scales!)**

  cross-helicity

  $\boldsymbol{U}\cdot\boldsymbol{B}$ **(for no-scale separation MHD)**

- ***Non-linear back reaction in dynamo***

  self-consistent models

The role of helicity in dynamo

Magnetic helicity

- inviscid invariant in MHD

$$H_m = \int (\mathbf{A}\cdot\mathbf{B}) d^3\mathbf{x}$$

Current helicity

$$H_C = \mu_0 \mathbf{B}\cdot\mathbf{j} = \mathbf{B}\cdot\nabla\times\mathbf{B}$$

observational proxy of mean magnetic helicity in solar active regions (Zhang et al. 2012)

$$\langle \mathbf{B}^{\mathrm{ar}}\cdot\mathrm{curl}\ \mathbf{B}^{\mathrm{ar}} \rangle \sim -\frac{1}{L_{\mathrm{ar}}^2}\langle\mathbf{A}\rangle\cdot\langle\mathbf{B}\rangle$$

- signature of the alpha-effect (Seehafer 1994)

$$\alpha \equiv \frac{\mathscr{E}\cdot\langle \boldsymbol{B}\rangle}{\langle \boldsymbol{B}\rangle^2} = -\frac{\eta}{\langle \boldsymbol{B}\rangle^2}\langle \boldsymbol{B}'\cdot\mathrm{curl}\,\boldsymbol{B}'\rangle$$

Dynamo model with evolution of Helicity

Magnetic field generation (Parker Dynamo)

$$\frac{\partial}{\partial t}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{bmatrix} \frac{\partial^2}{\partial \theta^2} & \alpha(\theta) \\ -D\cos\theta\frac{\partial}{\partial \theta} & \frac{\partial^2}{\partial \theta^2} \end{bmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\frac{\partial}{\partial t} H_c = f\left[H_c, \begin{pmatrix} A \\ B \end{pmatrix} \right]$$

Generation of Helicity

Parameterized equation *(Kleeorin, Ruzmaikin, 1982)*

Further development of the 1D model

(Moss,Kleeorin,Rogachevskii, Sokoloff, Kuzanyan et al.)

$$\frac{\partial A}{\partial t} = \alpha B + \frac{\partial^2 A}{\partial \theta^2} - \mu^2 A - V_\theta^M \frac{\partial A}{\partial \theta} ,$$

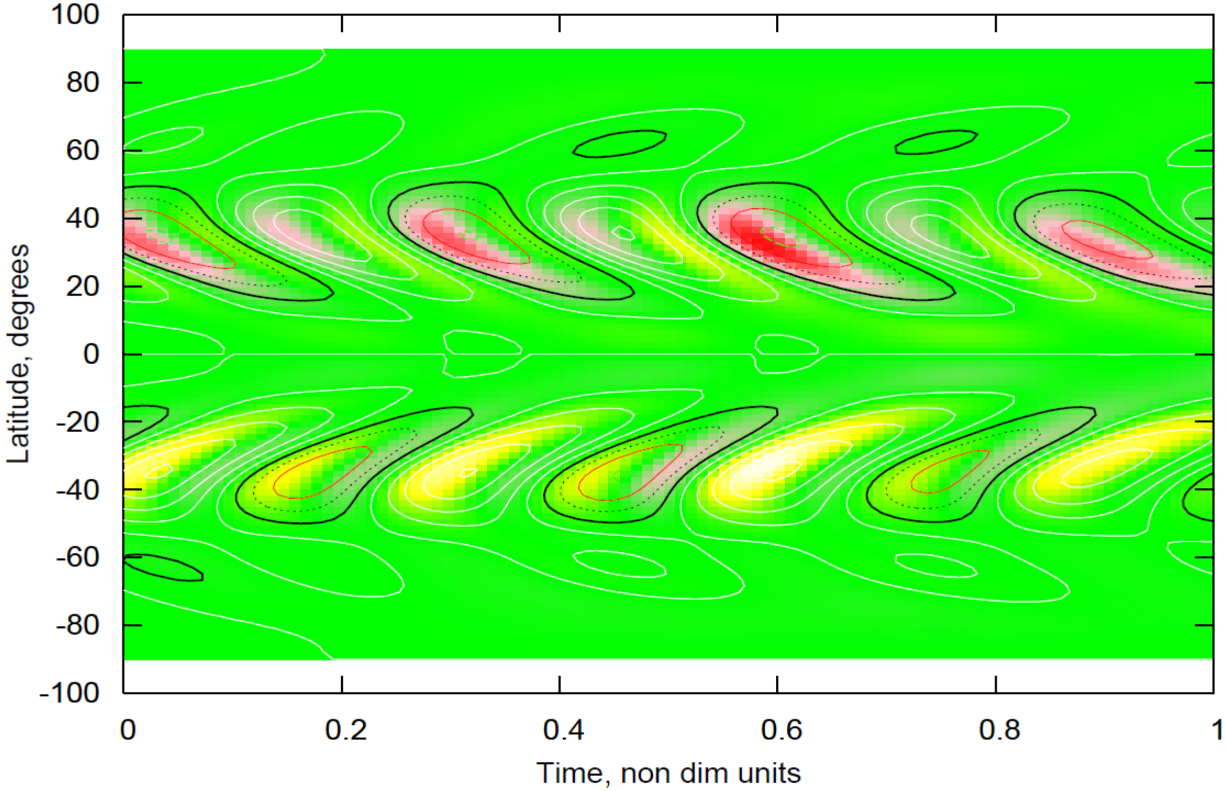
$$\frac{\partial B}{\partial t} = G_r D \sin\theta \frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2} - \mu^2 B - V_\theta^M \frac{\partial B}{\partial \theta} .$$

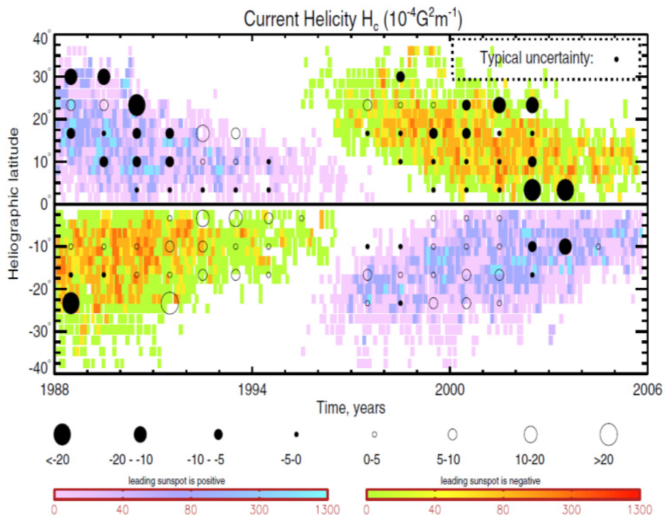
$$\alpha = \alpha^v + \alpha^m = \chi^v \phi_v + \phi_m \chi^c .$$!

$$\frac{\partial \chi^c}{\partial t} + (T^{-1} + \kappa\mu^2)\chi^c = \left(\frac{2R_\odot}{\ell}\right)^2 \left[\frac{\partial A}{\partial \theta}\frac{\partial B}{\partial \theta} - B\frac{\partial^2 A}{\partial \theta^2} - \alpha B^2 + 2\mu^2 AB + CB^2 \phi_v \chi^v(\theta)\right] + \kappa \frac{\partial^2 \chi^c}{\partial \theta^2} - \frac{\partial (V_\theta^M \chi^c)}{\partial \theta} , \qquad (4)$$

Estimate of current helicity of active regions

$$\langle \overline{\mathbf{B}} \cdot (\nabla \times \overline{\mathbf{B}}) \rangle \sim -\frac{1}{\eta_T \, T_c} \mathbf{A} \cdot \mathbf{B}$$





Zhang, Moss, Sokoloff, Kuzanyan, Kleeorin, Rogachevskii (2012)

2D dynamo model (2012)

$$\frac{\partial \tilde{A}}{\partial t} + \frac{(V_\theta^A + V_\theta^M)}{r}\frac{\partial \tilde{A}}{\partial \theta} + (V_r^A + V_r^M)\frac{\partial \tilde{A}}{\partial r} = C_\alpha\, \alpha\, \tilde{B} + \eta_A\left[\frac{\partial^2 \tilde{A}}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial \tilde{A}}{\partial \theta}\right)\right], \quad \text{(A1)}$$

$$\frac{\partial \tilde{B}}{\partial t} + \frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left[\frac{(V_\theta^B + V_\theta^M)\tilde{B}}{\sin\theta}\right] + \frac{\partial[(V_r^B + V_r^M)\tilde{B}]}{\partial r} = \sin\theta\left[G_r\frac{\partial}{\partial \theta} - G_\theta\frac{\partial}{\partial r}\right]\tilde{A} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial \theta}\left[\frac{\eta_B}{\sin\theta}\frac{\partial \tilde{B}}{\partial \theta}\right] + \frac{\partial}{\partial r}\left[\eta_B\frac{\partial \tilde{B}}{\partial r}\right], \quad \text{(A2)}$$

2D dynamo model (*cont.*)

The total α-effect is given by

$$\alpha = \alpha^v + \alpha^m = \chi^v \phi_v + \frac{\phi_m}{\rho(z)} \chi^c \,.$$

hydrodynamical part

$$\chi^v = \sin^2\theta \, \cos\theta$$

Magnetic part of the alpha-effect

$$\frac{\partial \tilde{\chi}^c}{\partial t} + \frac{\tilde{\chi}^c}{T} = \left(\frac{2R_\odot}{\ell}\right)^2 \left\{ \frac{1}{C_\alpha} \left[\frac{\eta_B}{r^2} \frac{\partial \tilde{A}}{\partial \theta} \frac{\partial \tilde{B}}{\partial \theta} + \eta_B \frac{\partial \tilde{A}}{\partial r} \frac{\partial \tilde{B}}{\partial r} \right. \right.$$

$$- \eta_A \tilde{B} \frac{\sin\theta}{r^2} \frac{\partial}{\partial \theta} \left(\frac{1}{\sin\theta} \frac{\partial \tilde{A}}{\partial \theta} \right) - \eta_A \tilde{B} \frac{\partial^2 \tilde{A}}{\partial r^2}$$

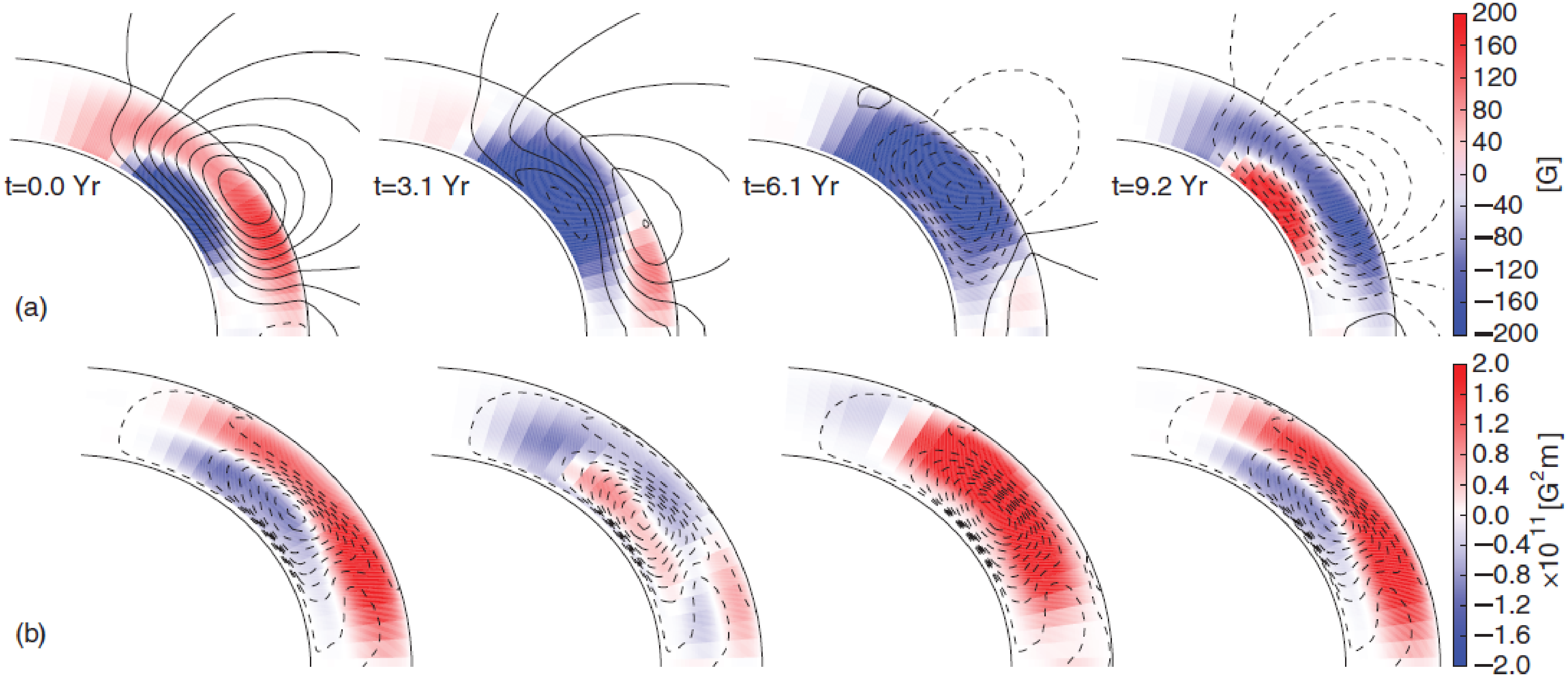
$$\left. \left. + (V_r^A - V_r^B) \tilde{B} \frac{\partial \tilde{A}}{\partial r} + (V_\theta^A - V_\theta^B) \frac{\tilde{B}}{r} \frac{\partial \tilde{A}}{\partial \theta} \right] - \alpha \tilde{B}^2 \right\}$$

$$- \frac{\partial (\tilde{\mathcal{F}}_r + V_r^M \tilde{\chi}^c)}{\partial r} - \frac{\sin\theta}{r} \frac{\partial}{\partial \theta} \left[\frac{\tilde{\mathcal{F}}_\theta + V_\theta^M \tilde{\chi}^c}{\sin\theta} \right], \quad \text{(A5)}$$

An estimate for the current helicity of active regions

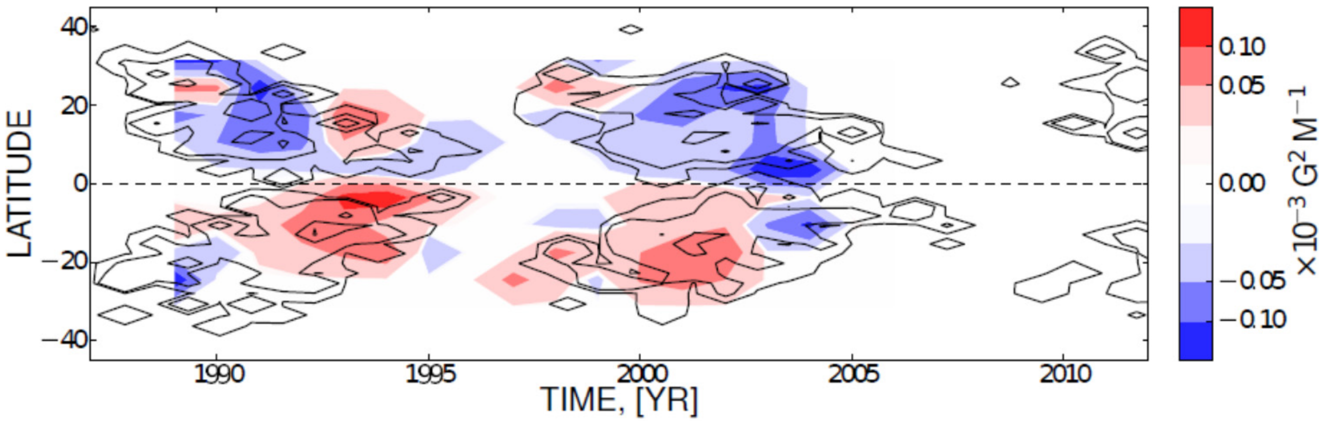
$$\langle \overline{\mathbf{B}} \cdot (\nabla \times \overline{\mathbf{B}}) \rangle \sim -\frac{1}{\eta_T T_c} \mathbf{A} \cdot \mathbf{B}$$

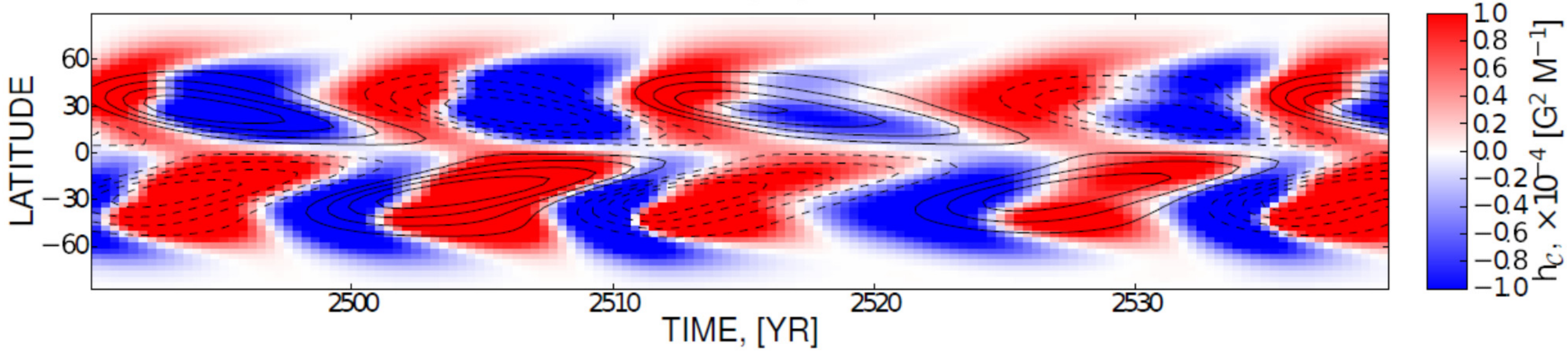
2D Dynamo Model with Total (small-scale + large scale) Magnetic Helicity Conservation



(Pipin, Sokoloff, Zhang, Kuzanyan 2013)

Results: 2D dynamo model with total helicity conservation

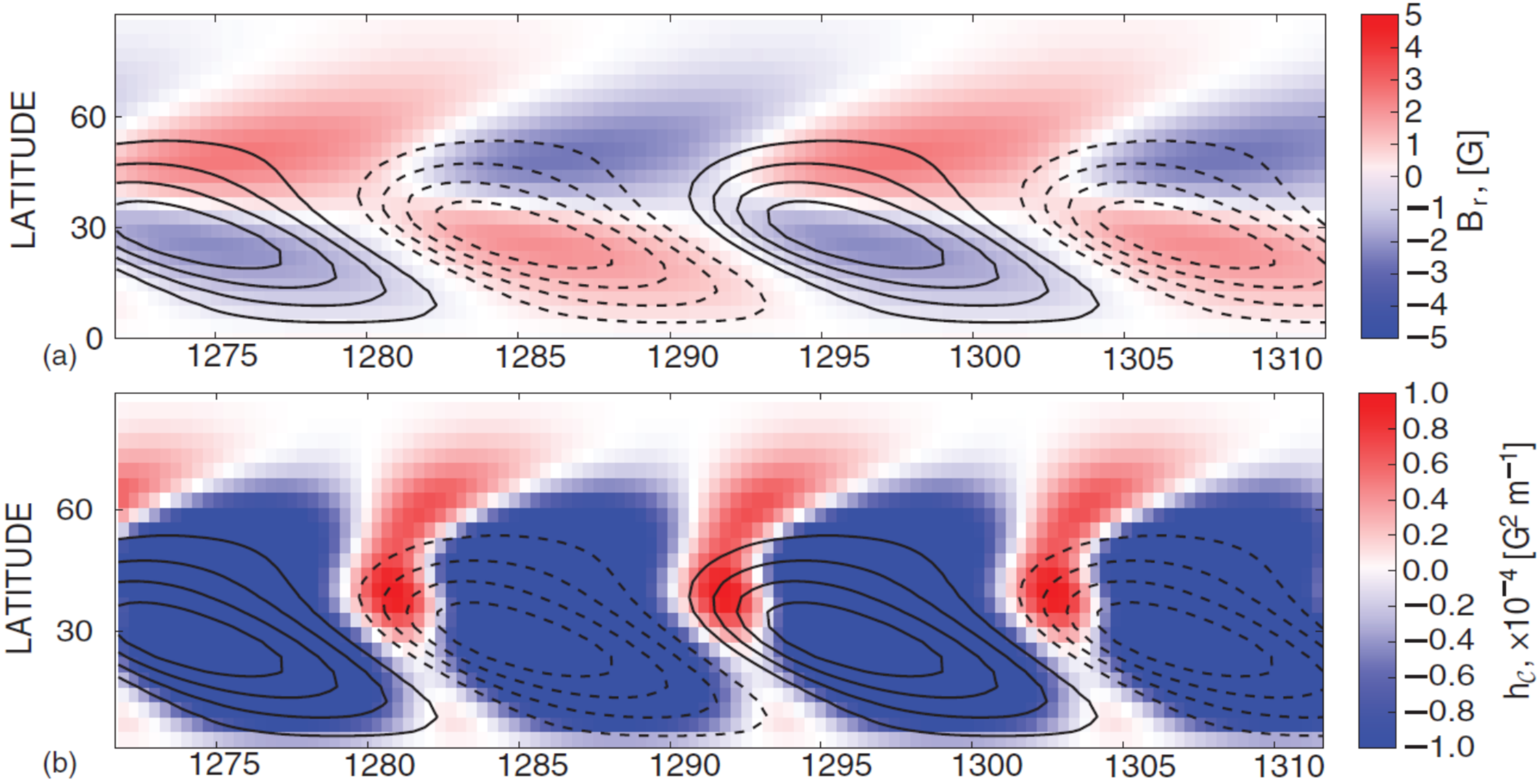




Magnetic field & current helicity: comparing the observations and the model

(Pipin, Zhang, Sokoloff, Kuzanyan, Gao 2013)

Results: 2D dynamo model with total helicity conservation



Magnetic field & current helicity: contour plot

(Pipin, Sokoloff, Zhang, Kuzanyan 2013)

2D model with cross-helicity

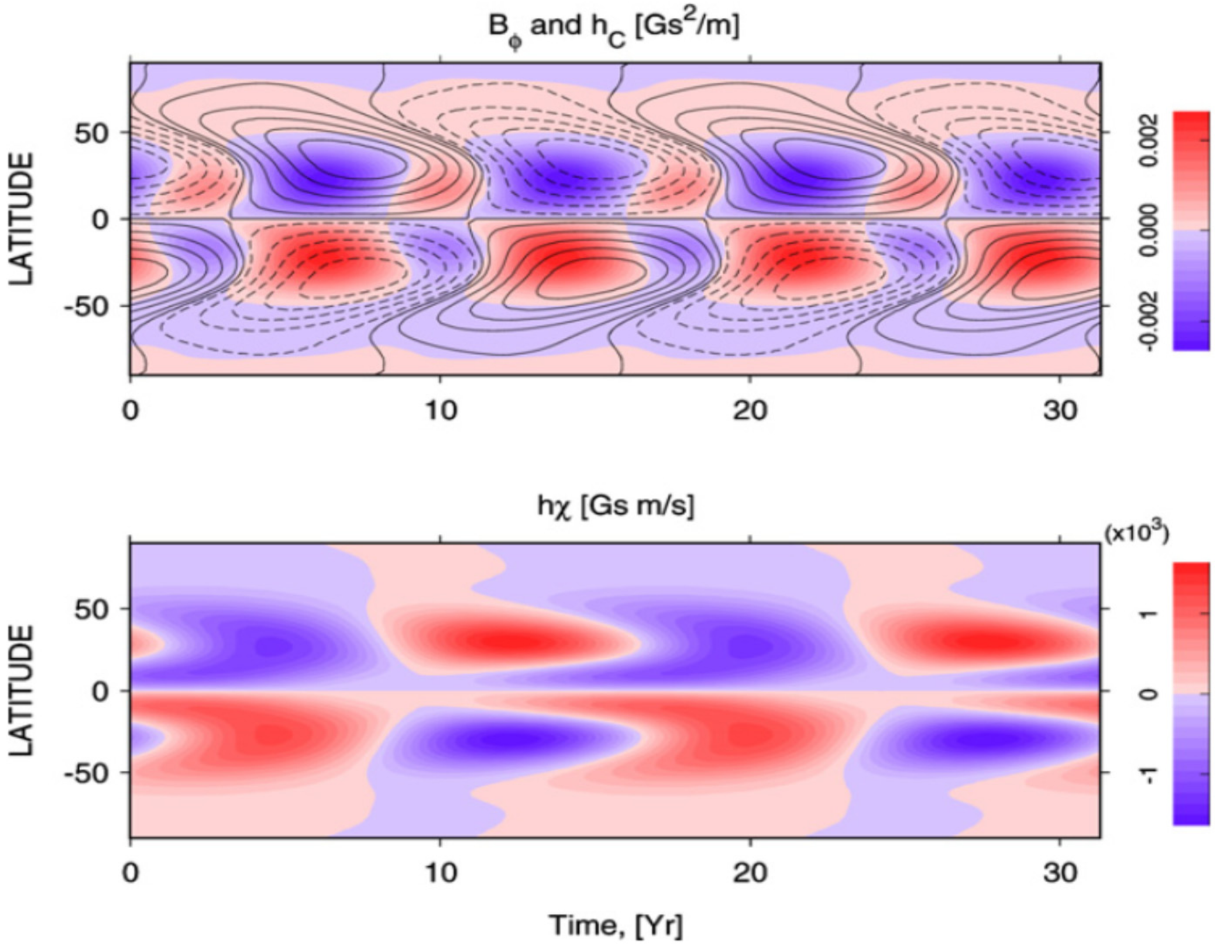
(after Pipin, Kuzanyan, Zhang & Kosovichev, 2011)

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times \left(\mathbf{u} \times \overline{\mathbf{B}} + \overline{\mathbf{U}} \times \mathbf{b} \right) + \eta \nabla^2 \mathbf{b} + \mathfrak{G} ,$$

$$\begin{aligned}
\frac{\partial m_i}{\partial t} &= -2 \left(\mathbf{\Omega} \times \mathbf{m} \right)_i - \nabla_i \left(p - \frac{2}{3} \left(\mathbf{G} \cdot \mathbf{m} \right) \nu + \frac{\left(\mathbf{b} \cdot \overline{\mathbf{B}} \right)}{2\mu} \right) \\
&+ \nu \Delta m_i + \nu \left(\mathbf{G} \cdot \nabla \right) m_i + f_i + \mathfrak{F}_i \\
&+ \frac{1}{\mu_0} \nabla_j \left(\overline{B}_j b_i + \overline{B}_i b_j \right) - \nabla_j \left(\overline{U}_j m_i + \overline{U}_i m_j \right) ,
\end{aligned}$$

$$\begin{aligned}
\partial_t \left(\overline{\mathbf{u} \cdot \mathbf{b}} \right) &= \frac{1}{\bar{\rho}} \left(\overline{\mathbf{B}}^p \cdot \nabla \right) \left(\kappa_1 \bar{\rho} \overline{u^2} + \kappa_2 \frac{\overline{b^2}}{2\mu_0} \right) \\
&- \alpha \left(\overline{\mathbf{B}}^t \cdot \overline{\mathbf{W}}^t \right) + \mu_0 \eta_T \left(2\mathbf{\Omega} + \overline{\mathbf{W}}^p \right) \cdot \overline{\mathbf{J}}^p + \ldots ,
\end{aligned}$$

Results: model with cross-helicity



Magnetic field (contour) & Current Helicity (color);
Cross-Helicity (colour) *(Kuzanyan, Pipin, Zhang 2007).*

supporting theory: Observations of Solar Magnetic fields

~20 years systematic monitoring of the solar vector magnetic fields in active regions taken at Huairou Solar observing station, China (1988-2006+)

More observations from Mitaka (Japan) and also Mees, MSFC (USA) etc., but only Huairou data systematically cover 20 years period.

OBSERVATIONS

Vector Magnetogramms of Solar Active Regions:

$$\mathbf{B} = \{B_x, B_y, B_z\}(x, y)$$

vertical component of current $j_z = (\nabla \times \mathbf{B})_z$

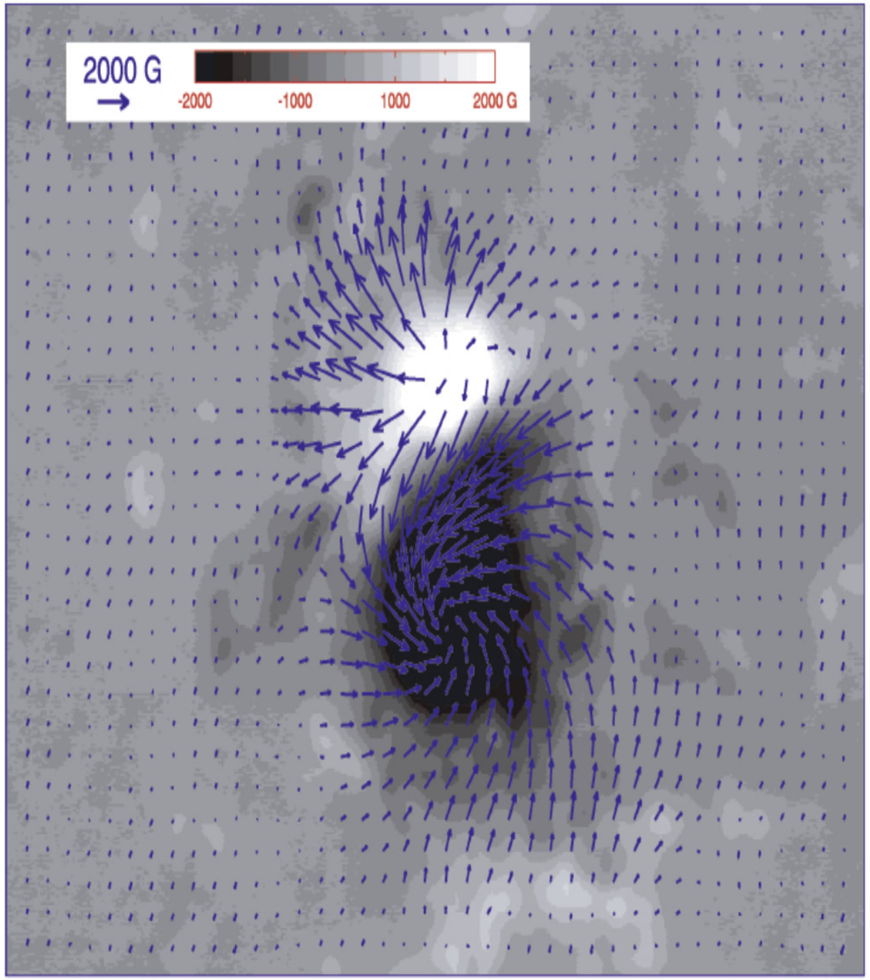
Calculation of Current Helicity $H_c = \mathbf{B} \cdot \nabla \times \mathbf{B} =$

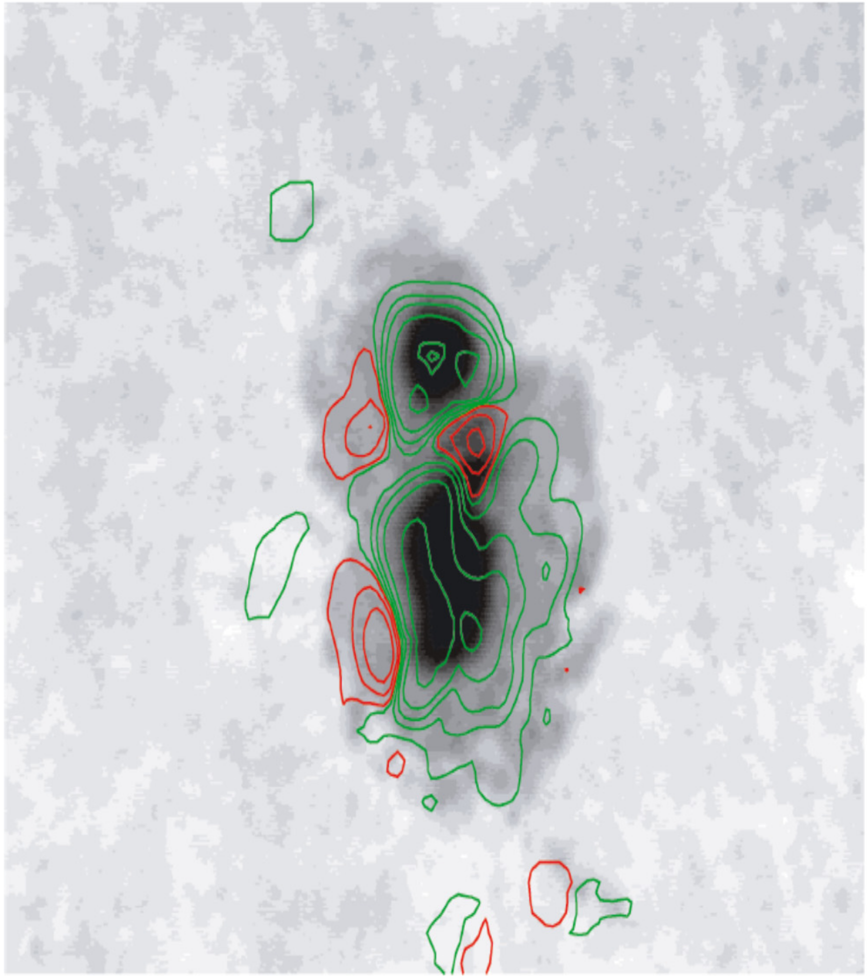
$$B_x(\nabla \times \mathbf{B})_x + B_y(\nabla \times \mathbf{B})_y + B_z(\nabla \times \mathbf{B})_z \, .$$

Twist $\gamma = \dfrac{\mathbf{B} \cdot \nabla \times \mathbf{B}}{\mathbf{B}^2}$

Observable !

AR NOAA6619 on 1991-5-11 @ 03:26UT (Huairou)





Photosphetic vector magnetogram

Current helicity over filtergram

Data Reduction

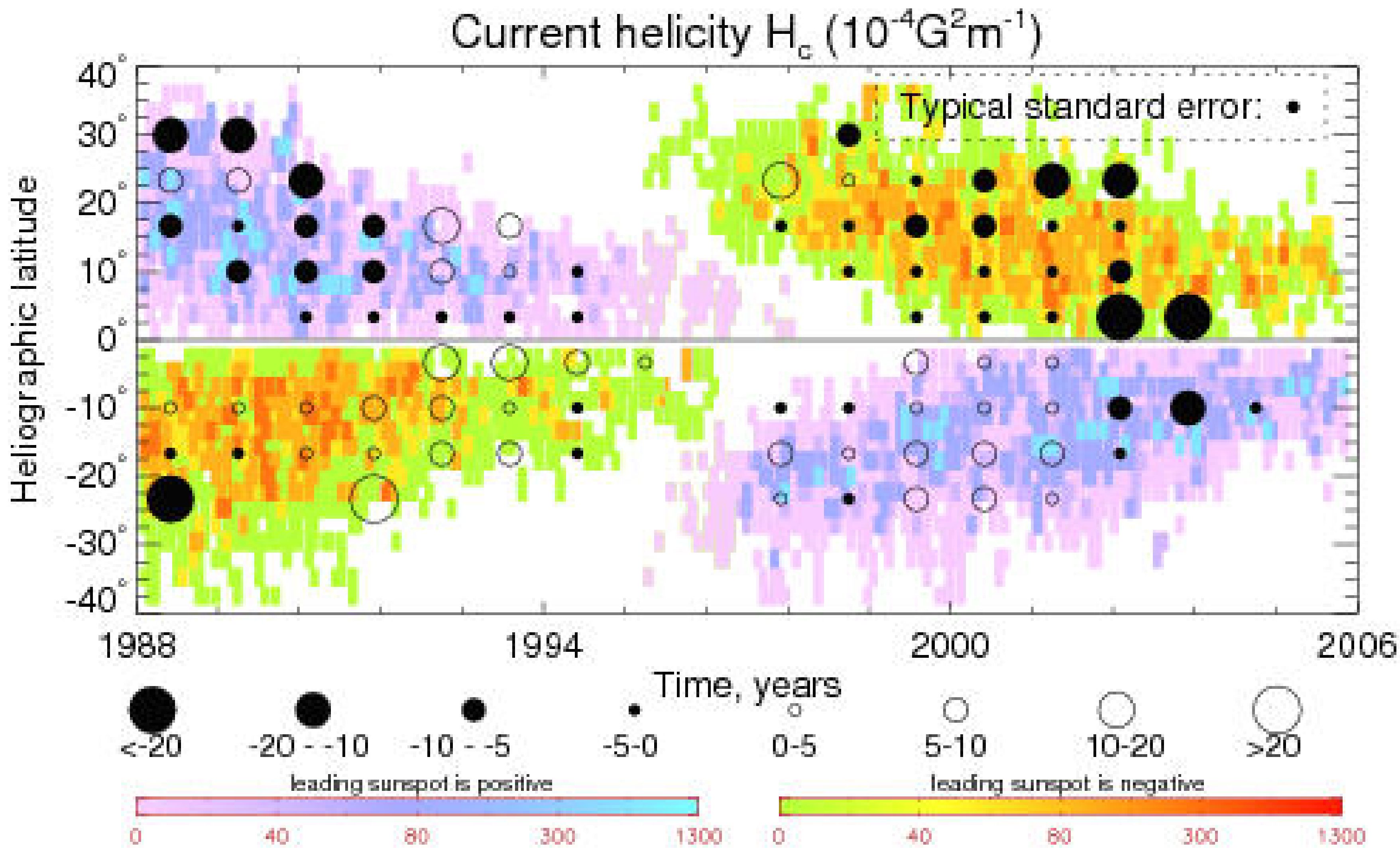
- **983 active regions; 6630 vector magnetograms observed at Huairou Solar Observing Station;**
- **Time average: 2 year bins (1988-2005);**
- **Latitudinal average: 7° bins;**

So, each bin contains 30+ magnetograms =>

=> independent statistics in each bin: compute averages with confidence intervals (*Student t* distribution) – see Zhang et al. (2010)

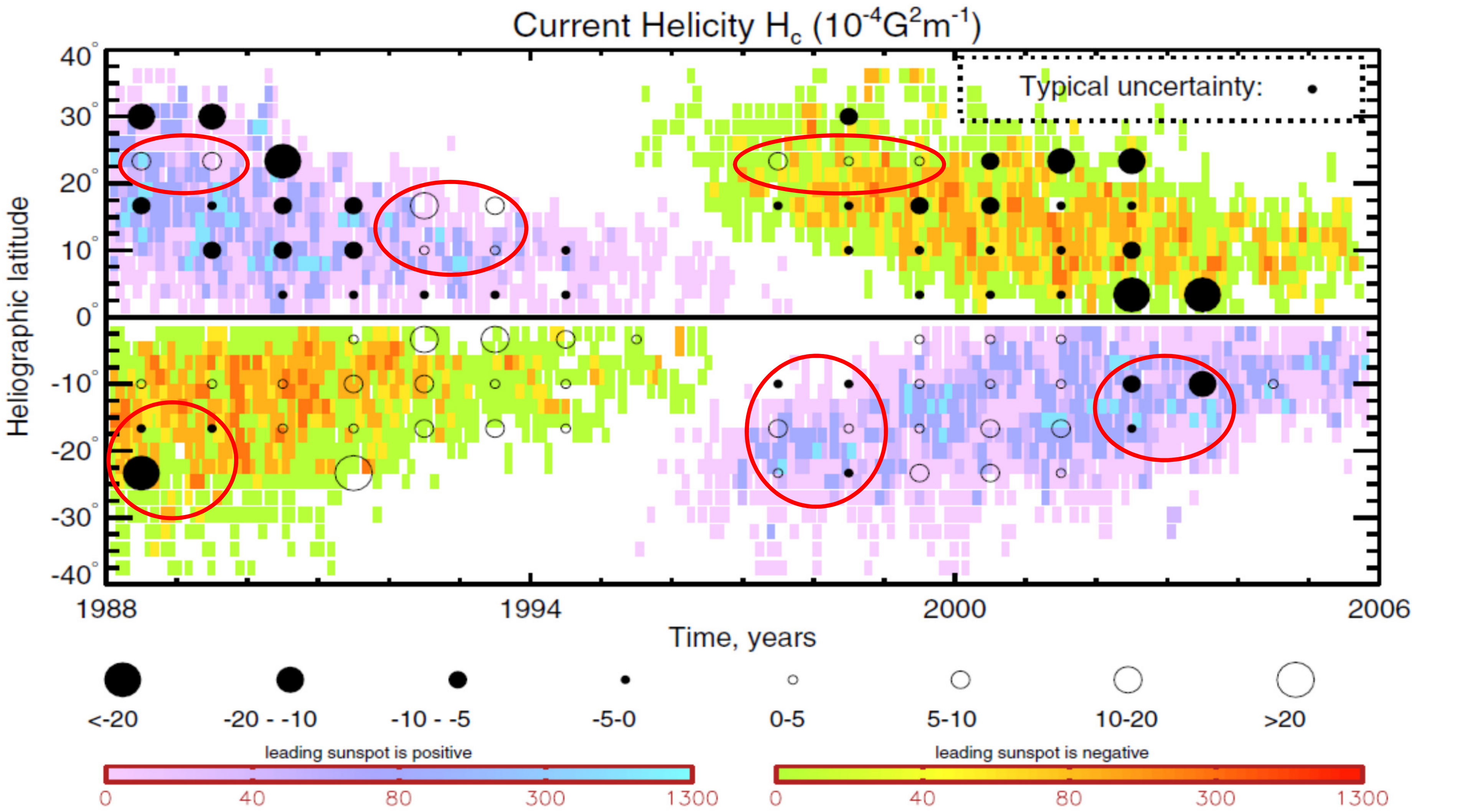
We assume the data subsamples equivalent to ensembles of turbulent pulsations, so we gather <u>mean</u> quantities in the sense of dynamo theory

Helicity overlaid with butterfly diagram



Helicity over the solar cycle:

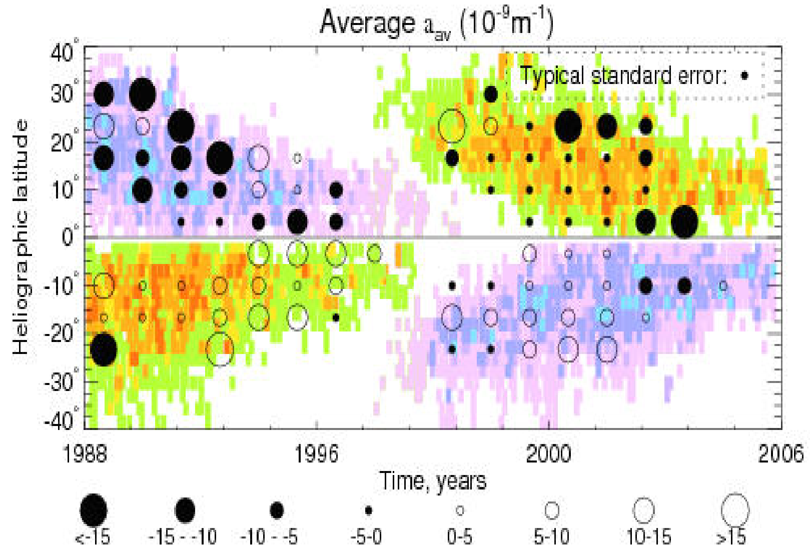
Zhang et al. (2010-2012)

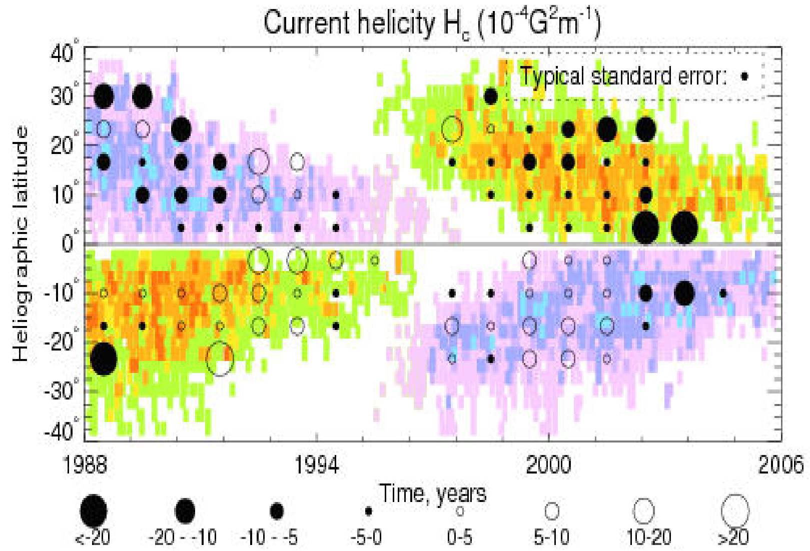


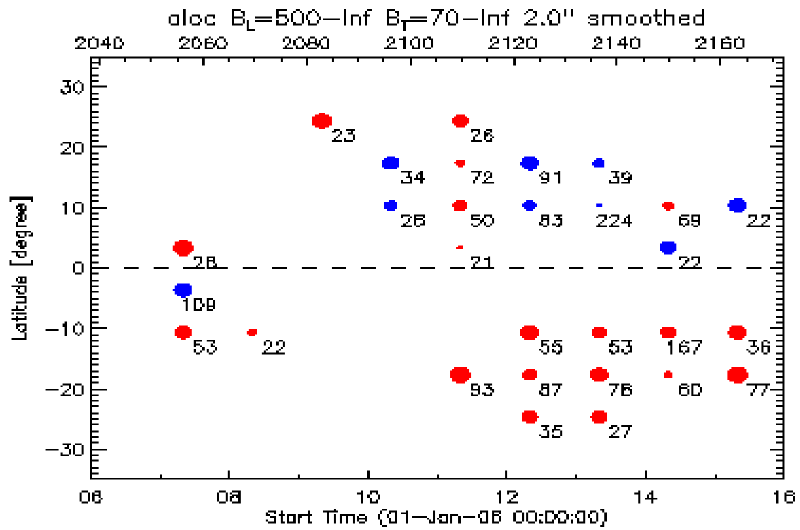
Important observational properties of helicity:

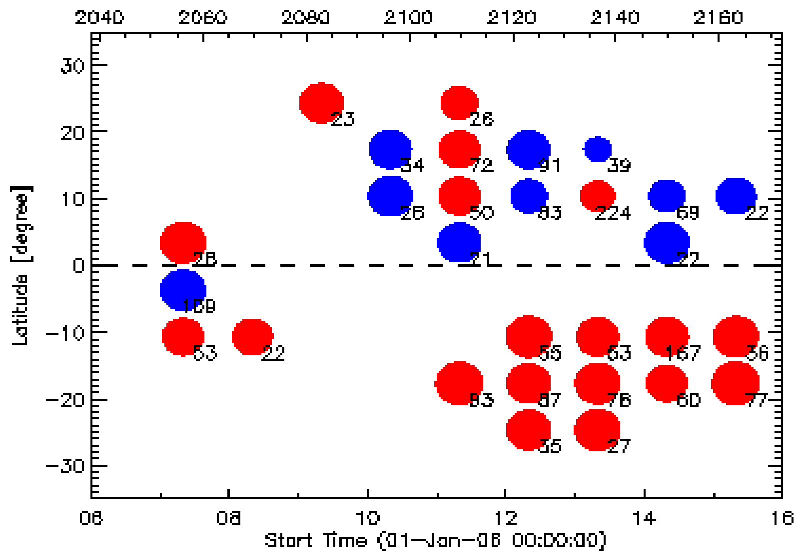
- Hemispheric Sign Rule:
  North=negative;
  South=positive
- Systematic reversal of the sign at some latitudes in the beginning and end of the solar cycle

Current Helicity and Twist in solar cycles 22-24 from Hinode vs. past data from ground









Otsuji, Sakurai, Kuzanyan (2015, PASJ)

OBSERVATIONS

Vector Magnetogramms of Solar Active Regions:

$$\mathbf{B} = \{B_x, B_y, B_z\}(x, y)$$

vertical component of current $j_z = (\nabla \times \mathbf{B})_z$

Calculation of Current Helicity $H_c = \mathbf{B} \cdot \nabla \times \mathbf{B} =$

$$B_x(\nabla \times \mathbf{B})_x + B_y(\nabla \times \mathbf{B})_y + \underline{\underline{B_z(\nabla \times \mathbf{B})_z}}\,.$$

Observable !

Twist $\gamma = \dfrac{\mathbf{B} \cdot \nabla \times \mathbf{B}}{\mathbf{B}^2}$

Question

- ***How the part of current helicity is really related to the entire quantity???***

  : How good is local homogeneity approximation?

(keep in mind!)

Definition of current helicity

$$\mathbf{B} \cdot (\nabla \times \mathbf{B})$$

$$= \mathrm{trace} \begin{pmatrix} B_x J_x & B_x J_y & B_x J_z \\ B_y J_x & B_y J_y & B_y J_z \\ B_z J_x & B_z J_y & B_z J_z \end{pmatrix}$$

where $\mathbf{J} = \nabla \times \mathbf{B}$.

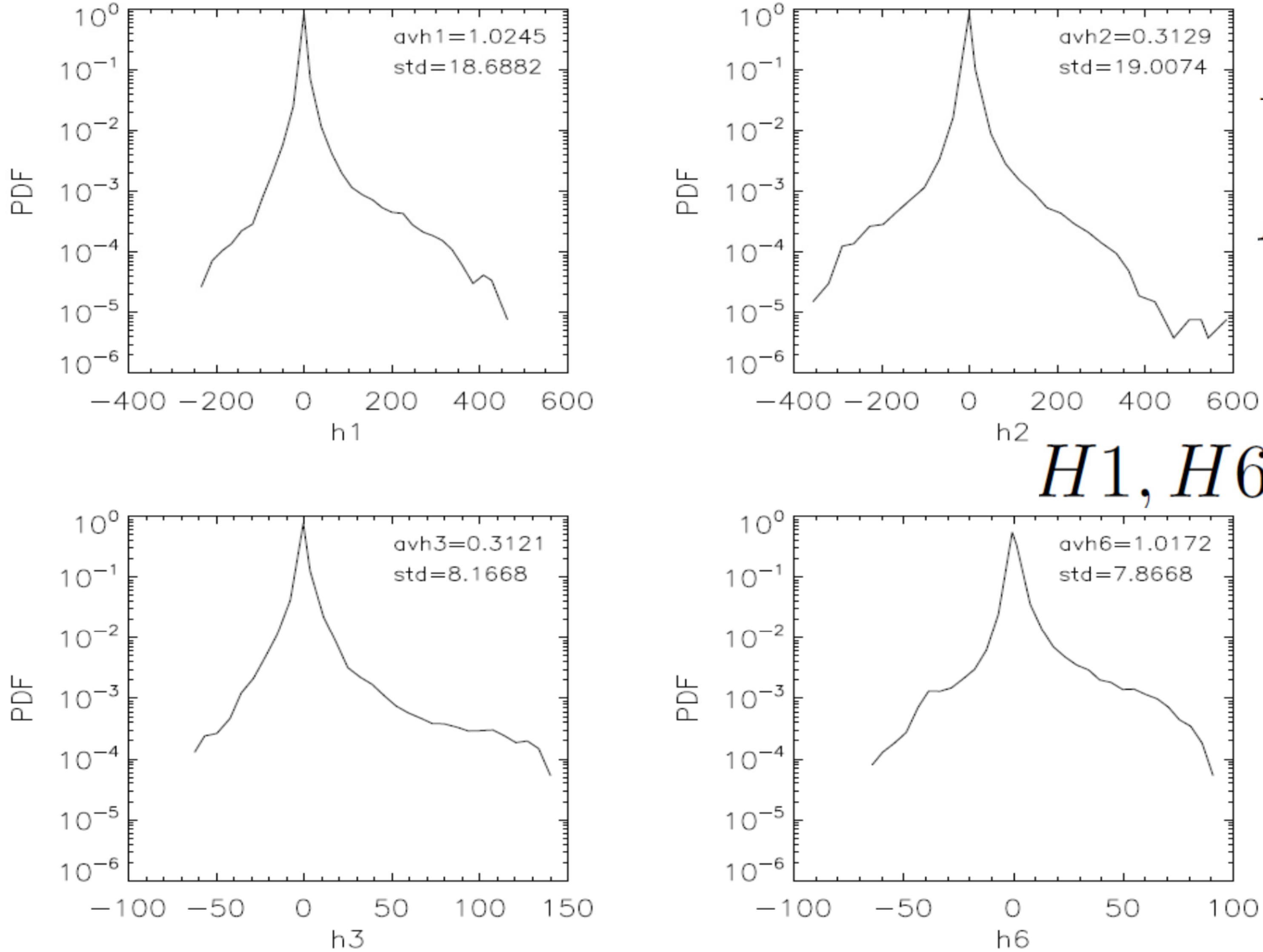
Definition of curl for any vector $\boldsymbol{F}$

$$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\mathbf{k}$$

Decomposition of current helicity into six parts:

$$\begin{aligned}
H_c &= \int \mathbf{J}\cdot\mathbf{B}dxdy = H1 + H2 + H3 + H4 + H5 + H6 \\
&= \int B_z\left(\frac{\partial B_y}{\partial x}\right)dxdy + \int B_z\left(-\frac{\partial B_x}{\partial y}\right)dxdy \\
&+ \int B_x\left(\frac{\partial B_z}{\partial y}\right)dxdy + \int B_x\left(-\frac{\partial B_y}{\partial z}\right)dxdy \\
&+ \int B_y\left(\frac{\partial B_x}{\partial z}\right)dxdy + \int B_y\left(-\frac{\partial B_z}{\partial x}\right)dxdy
\end{aligned}$$

Example of PDF for helicities (hr)

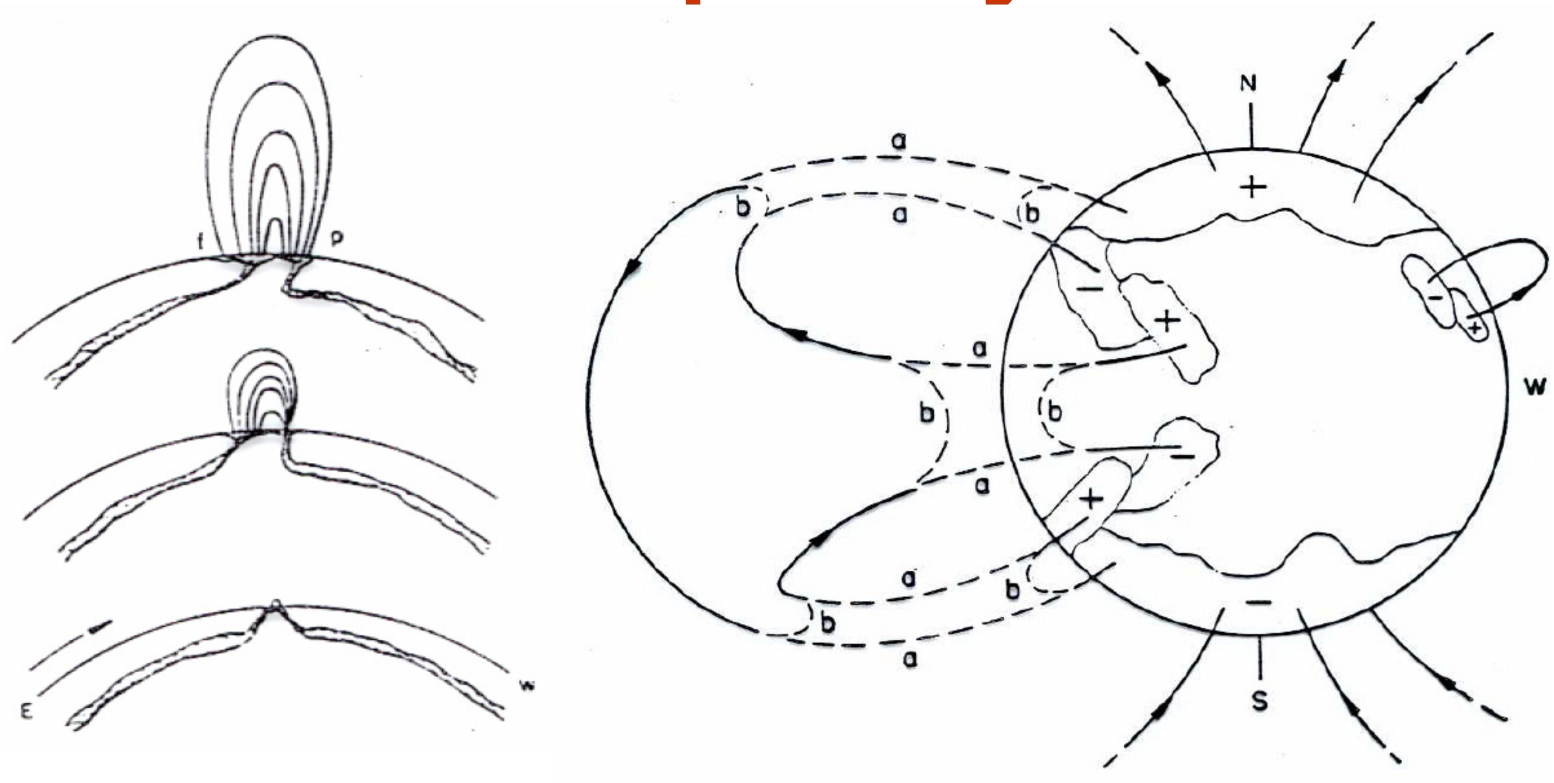


$$H1 \approx H6$$

$$H2 \approx H3$$

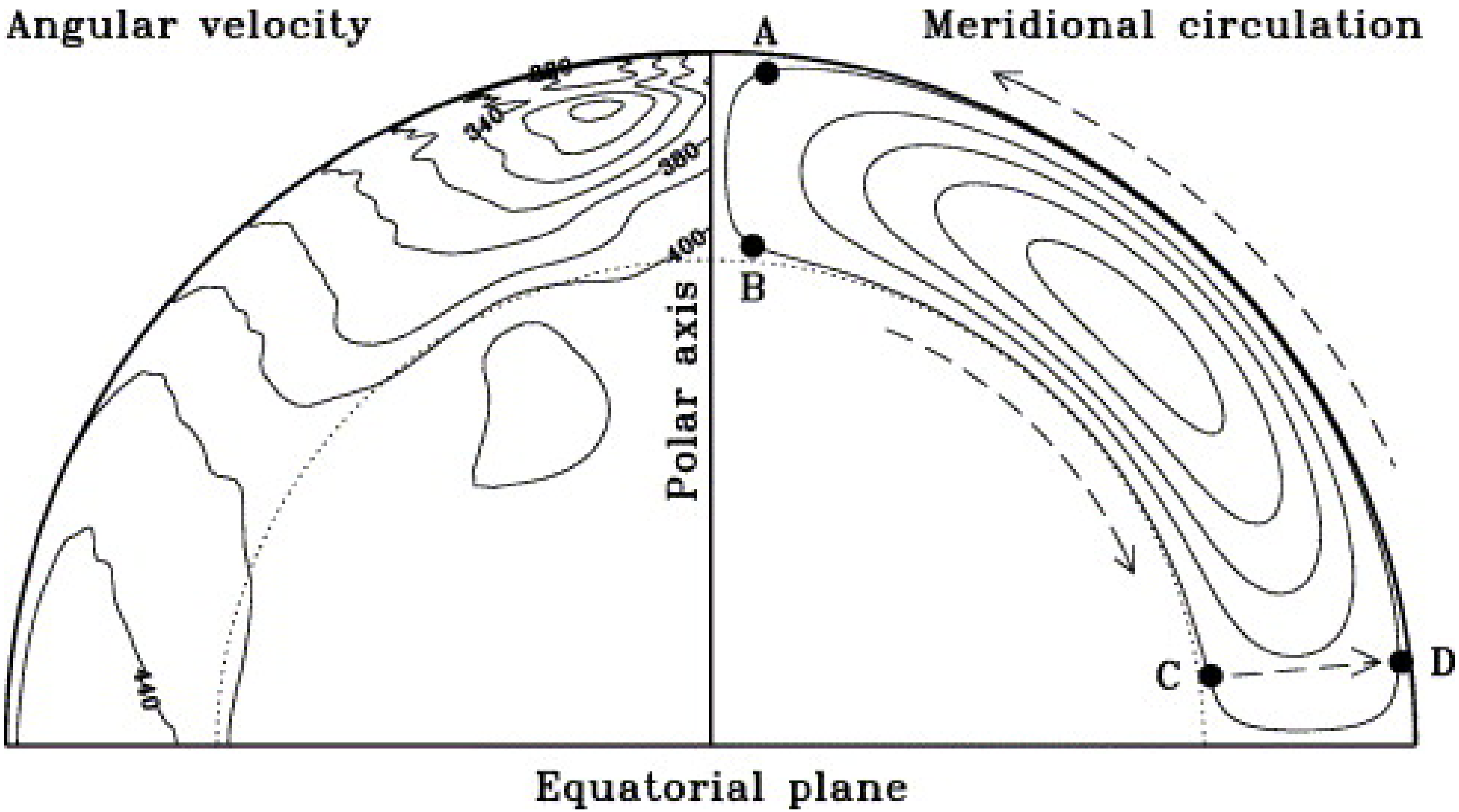
$$H1, H6 \neq H2, H3$$

The idea of Babcock & Leighton 1961-69:
“flux transport dynamo”

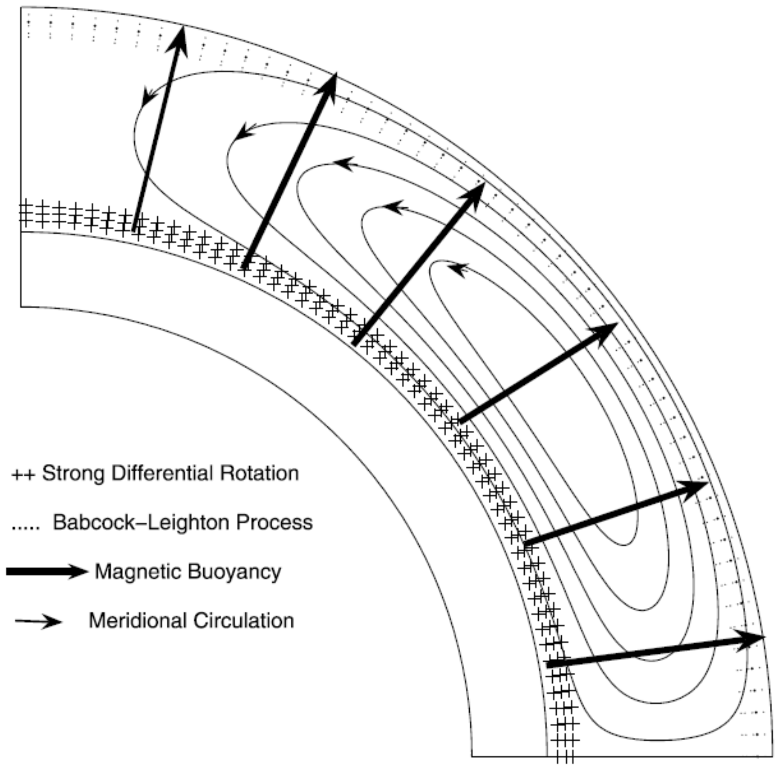


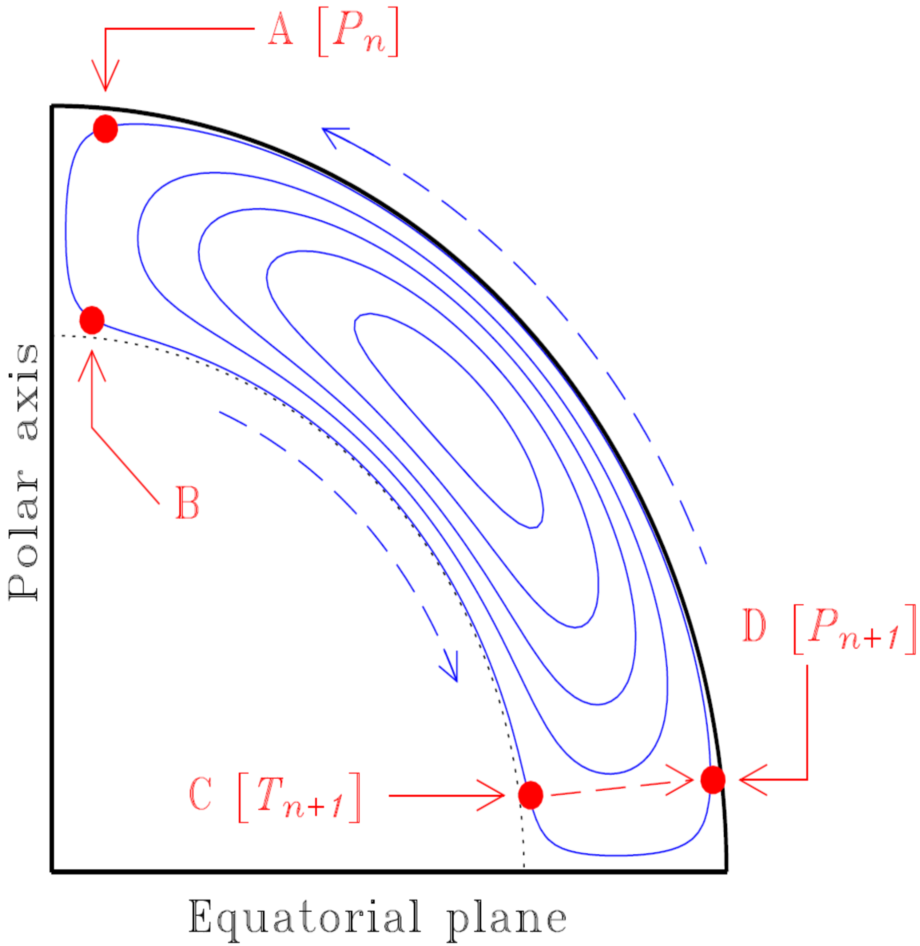
Babcock 1961, ApJ, 133, 572

The use of unknown
<u>meridional circulation</u>



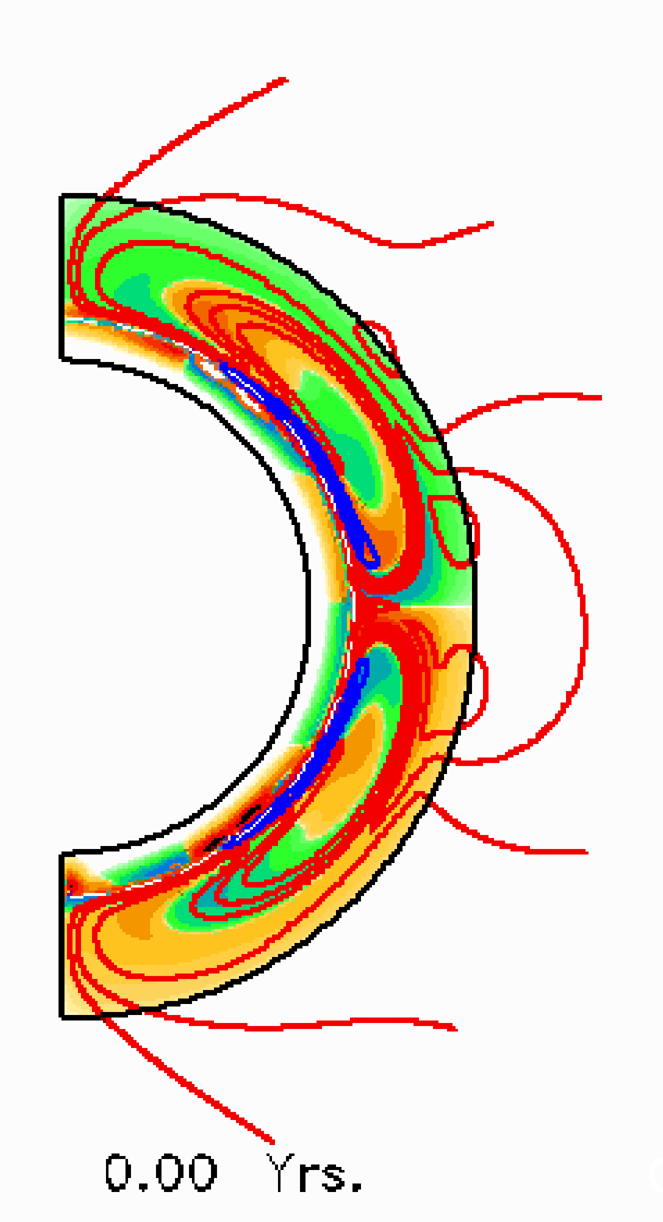
Babcock&Leighton Dynamo Mechanism



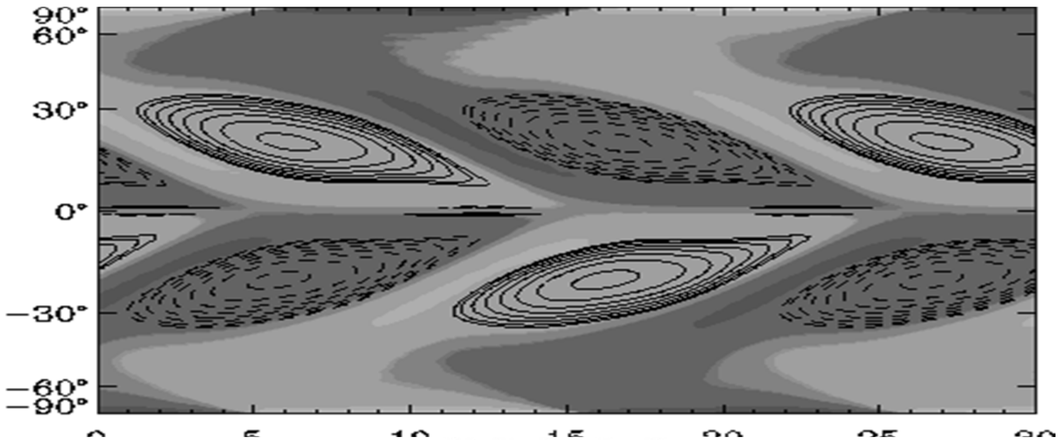


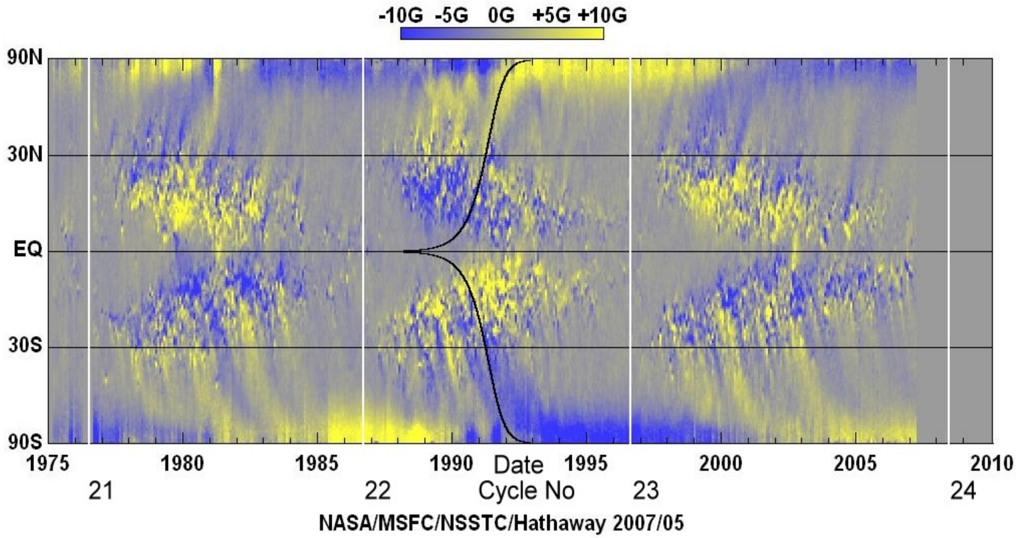
from Charbonneau (2010) and Karak et al. (2014)

Solar dynamo model



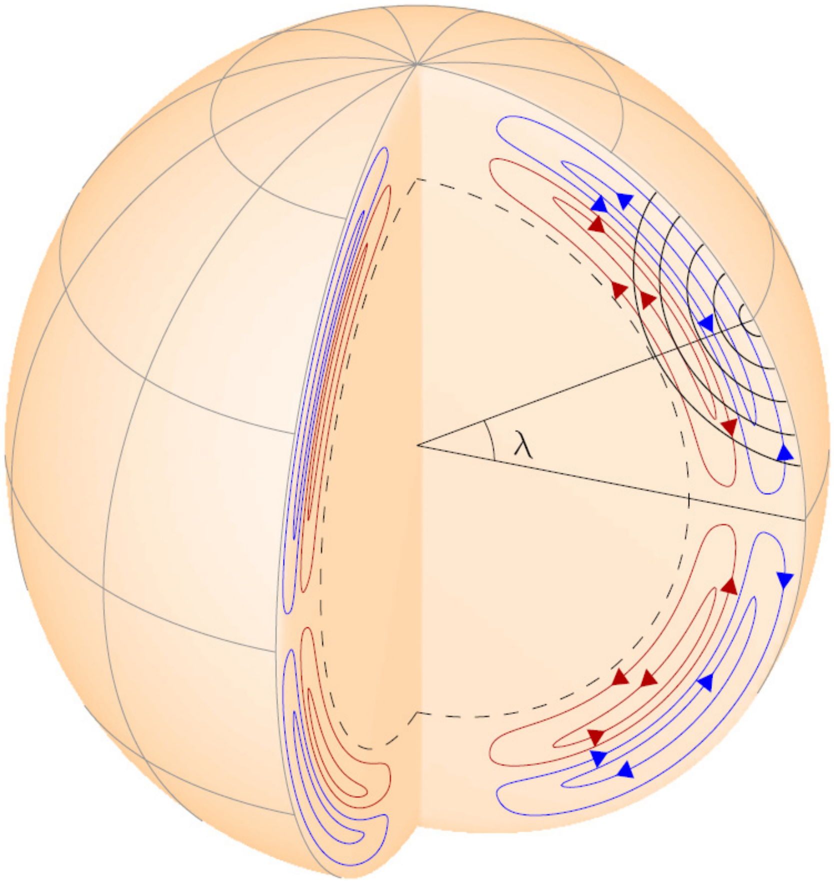
Contours: toroidal fields at CZ base
Gray-shades: surface radial fields

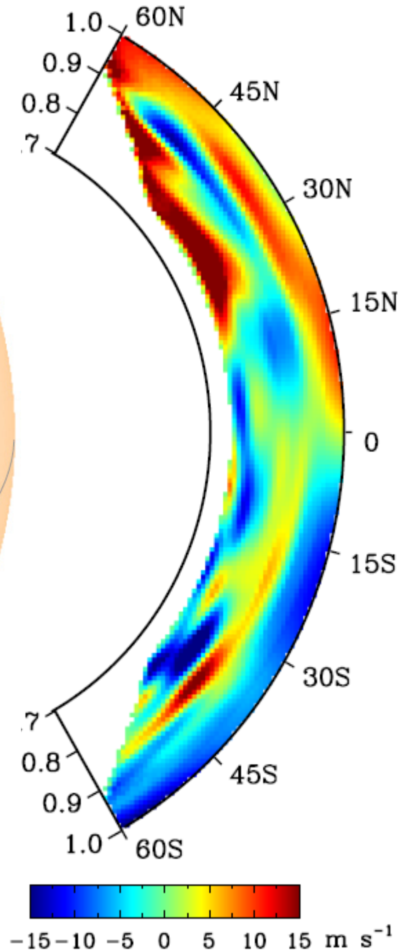


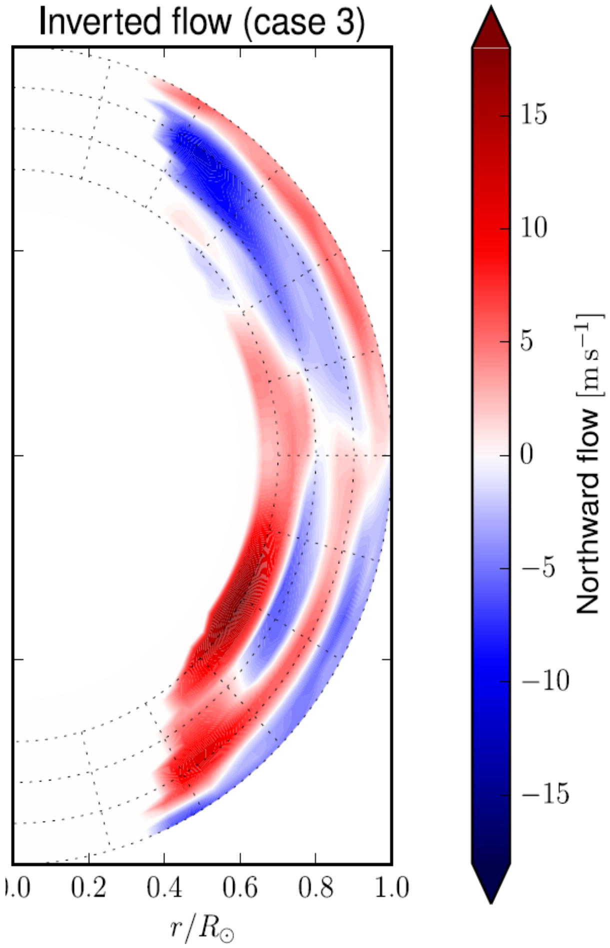


(Dikpati, de Toma, Gilman, Arge & White, 2004, ApJ, 601, 1136)

Recent helioseimology results: Double (Multiple-) cell meridional circulation

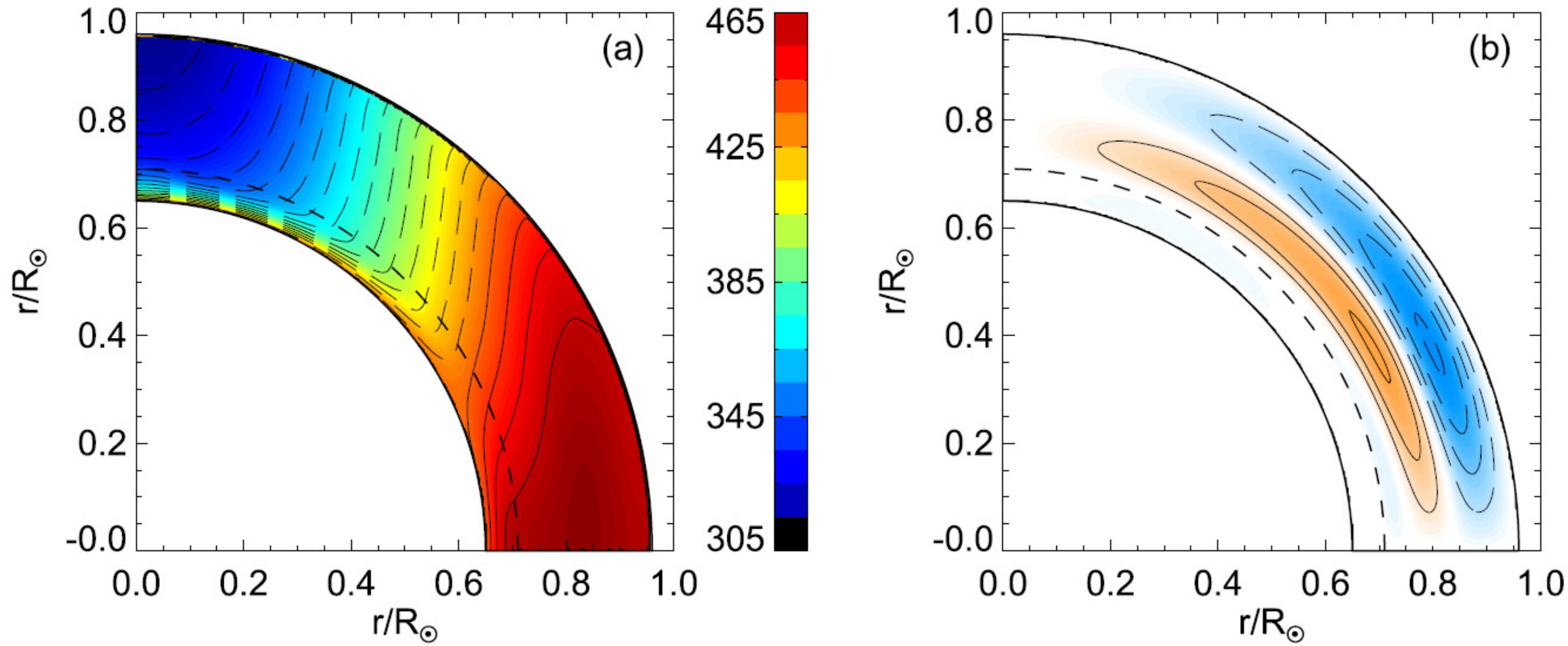






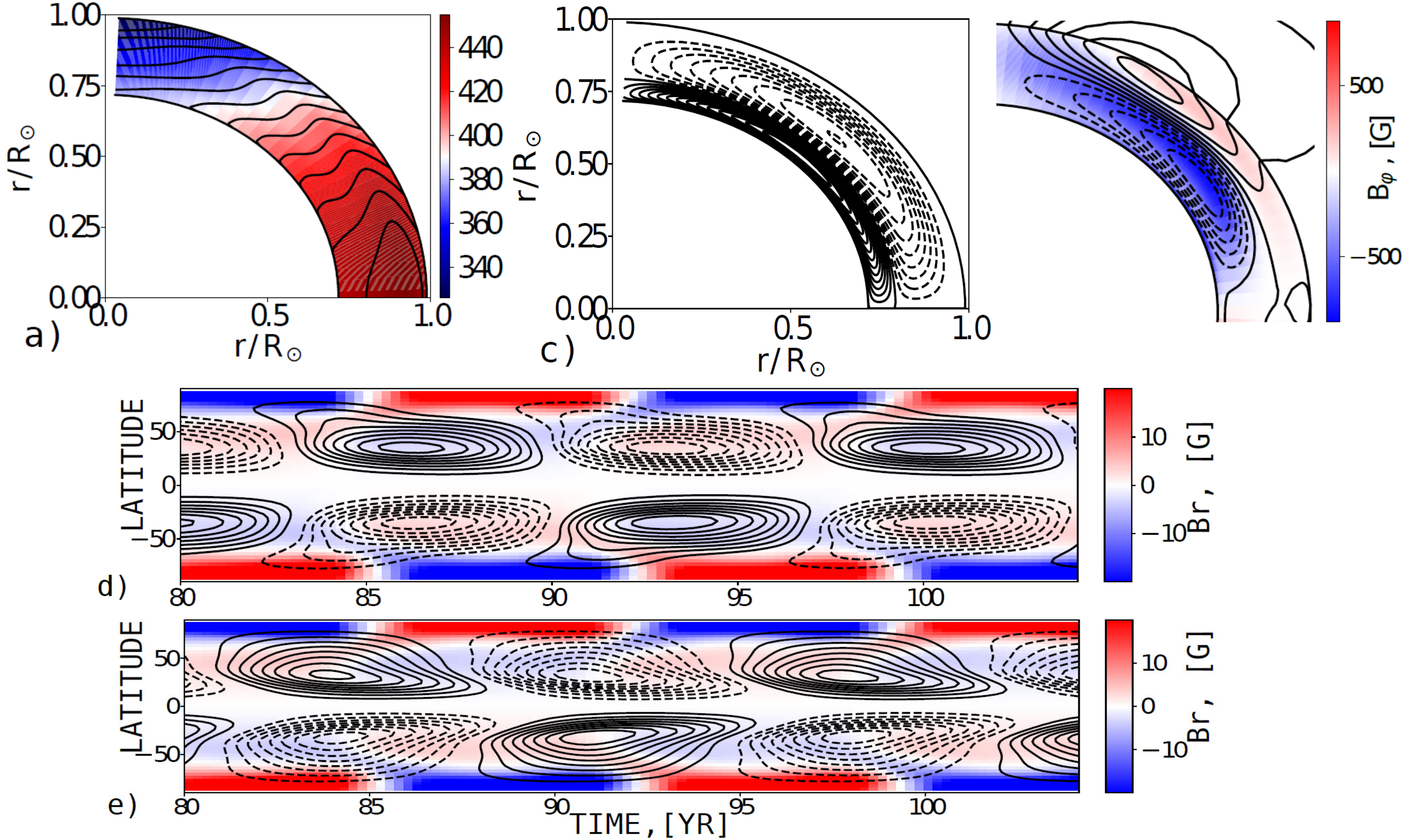
from Zhao et al. (2013) and Böning et al. (2017)

Self-consistent models dynamo+rotation+flow

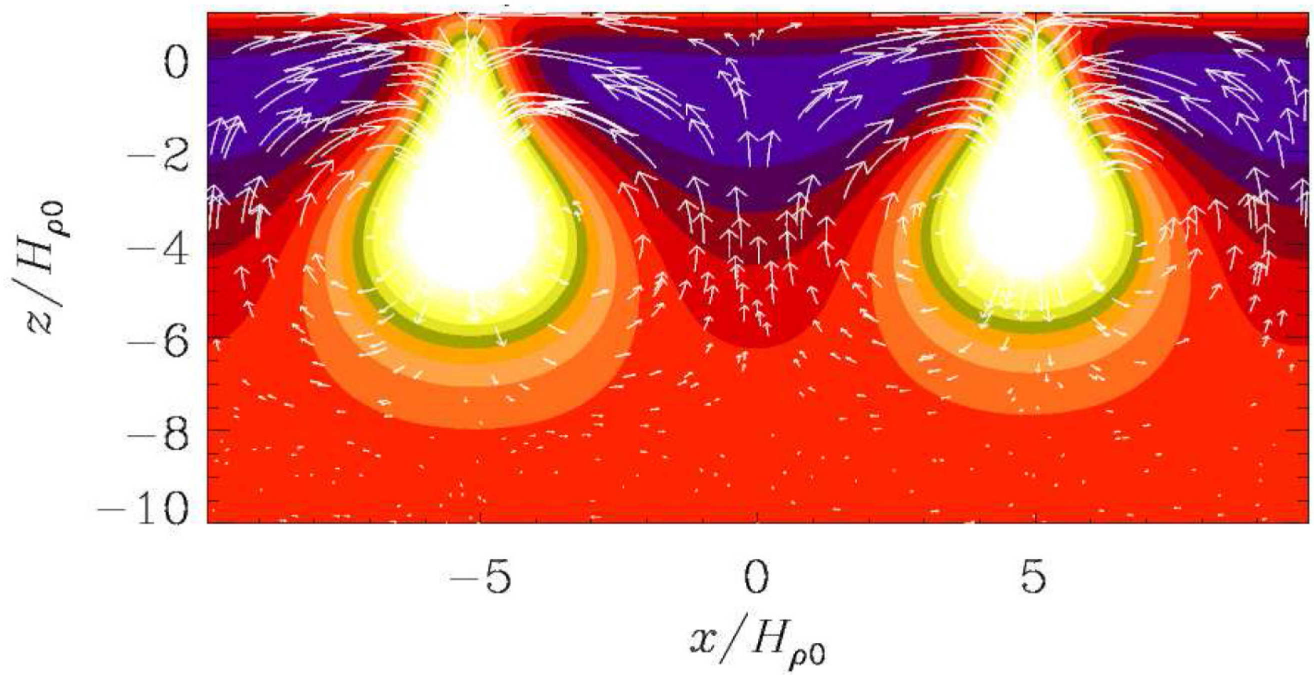


Bekki & Yokoyama, 2017

Self-consistent model by Pipin (2018)



Sunspot formation (NEMPI effect)



Sizes

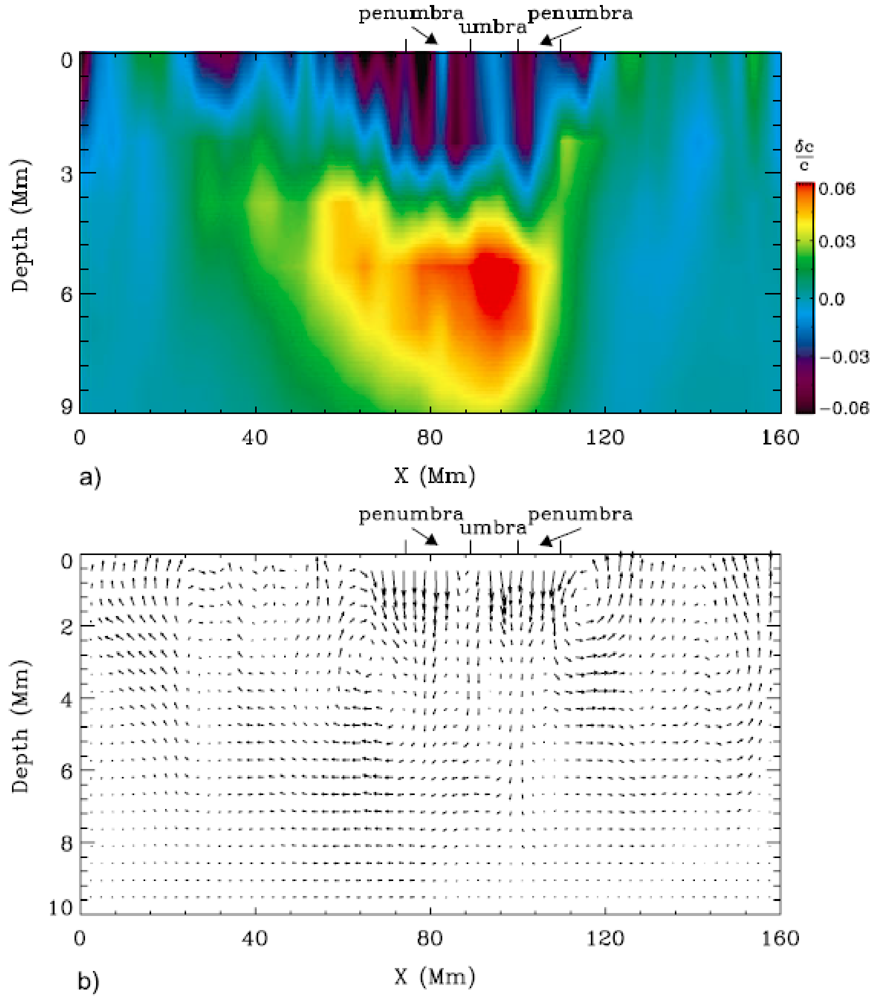
H x L

~5 x 10

pressure scales at surface

Formation of a sunspot from mean magnetic fields due to effect of Negative Effective Magnetic Pressure Instability(NEMPI) *Kleeorin +.1989-90*; series of works by *Brandenburg, Kleeorin, Rogachevskii (2010-2016)*; see the review *(New J. Phys. 18, 125011; 2016)*

Shallow flux tubes of sunspots



e.g. Kosovichev 2012

СПАСИБО!

谢谢！