Bayesian analysis of Konus-Wind solar flare data

Sergey Anfinogentov[1]

[1]Institute of solar-terrestrial physics

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The language of probability

A cheat sheet

The language of Physics	The Probability language
x could be anything	Flat distribution $P(x) = 1$
x is positive	Half flat distribution : $P(x) = \begin{cases} 1, & x > 0 \\ 0, & x \leq 0 \end{cases}$
x lies between a and b	Uniform distribution: $P(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$
According to the measurements: $x = x_0 \pm 3\sigma$	Normal distribution: $P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-x_0)^2}{2\sigma^2})$
Poison distribution: A device detected n photons in 1 second exposure. The photon flux through the device is $\lambda = n \pm \sqrt{n}$ (for large N)	The probability to observe n photons if λ us known $P(n\|\lambda) = \frac{e^{-\lambda}\lambda^n}{n!}$

Two approaches of probability interpretation

Frequentist approach	Bayesian approach
How frequent will the result appear in repetitive experiments?	What is the *degree of our belief* in the obtained result?
We expect to see 50 heads and 50 tails after flipping a fair coin 100 times.	After flipping a coin 100 times and observing 54 heads and 46 tails we are 90% sure that the coin is fair.
If a true value of a quantity is x_0, many measurement of it will be distributed by $P(x\|x_0) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{(x-x_0)^2}{2\sigma^2})$	If we have a single measurement of x and know σ, our knowledge about true value x_0 is $P(x_0\|x) \sim \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{(x-x_0)^2}{2\sigma^2})$
Forward problem	**Inverse problem**

The Bayes theorem

The knowledge about parameters $\boldsymbol{\theta = [\theta_1, \theta_2, \cdots, \theta_N]}$ of a model $\boldsymbol{M}$ is improved by the new data $\boldsymbol{D}$:

$$\boldsymbol{P(\theta|D, M) = \frac{P(D|\theta, M)P(\theta|M)}{P(D|M)}} \tag{1}$$

- $\boldsymbol{P(\theta|M)}$ – prior distribution (before seeing the data)
- $\boldsymbol{P(D|\theta|M)}$ – the likelihood function(information from the data)
- $\boldsymbol{P(\theta|D, M)}$ – Posterior distribution (improved knowledge)
- $\boldsymbol{P(D|M)}$ – Evidence of the model $\boldsymbol{M}$ (normalisation coefficient)

Model comparison

Probabilities of competing models $M_i = M_1, M_2 ... M_N$ can be calculated using the Bayes theorem:

$$P(M_i|D) = \frac{P(D|M_i)P(M_i)}{P(D)} \tag{2}$$

- $P(M_i)$– prior probability for a model M_i
- $P(D) = \sum_{j=1}^{N} P(D|M_j)P(M_i)$ – normalization constant

Bayes factor

The normalisation constant in $P(D|M)$ (1) from the Bayes theorem

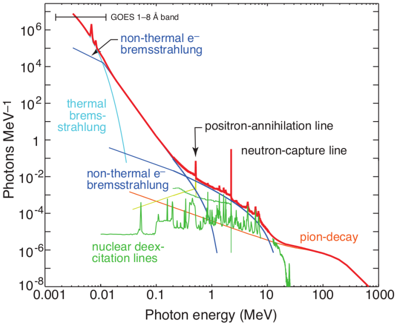
$$Z = P(D|M) = \int P(D|\theta, M)P(\theta|M)d\theta \quad (3)$$

It is a measure of how consistent with the data D is the model M .
Two models M_1 and M_2 can be quantitatively compared by calculating the Bayes factor:

$$B_{12} = \frac{P(D|M_1)}{P(D|M_2)} \quad (4)$$

B_{12}	$2\ln B_{12}$	Evidence towards model 1	Prob. of model 1
$1-3$	$0-2$	Barely worth mentioning	$0.5-0.75$
$3-20$	$2-6$	positive	$0.75-0.95$
$20-150$	$6-10$	strong	$0.95-0.99$
>150	>10	very strong	>0.99

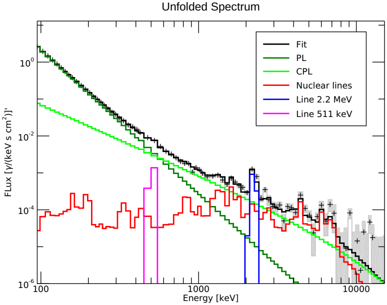
Solar flare spectrum in gamma-ray range

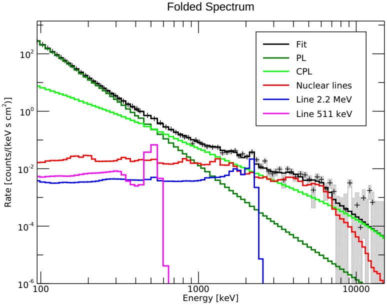


Credits: Ronald Murphy

- Bremsstrahlung continuum from accelerated electrons and positrons:
- Components caused by accelerated ions are results of nuclear reactions:
    - Nuclear deexcitation lines (templates): nuclear transitions from excited to ground state.
    - Electron-positron annihilation line at 511 keV (gaussian line) from positrons born in β^+-decay or decay of π^+.
    - Neutron capture line p+n $\rightarrow$ ^{2}H+$\gamma_{2.223\,MeV}$.
    - Continuum from π^0 decay – outside Konus-*Wind* spectral range.

Konus-Wind observation of an X9.3 flare[1] detected on 2017-09-06

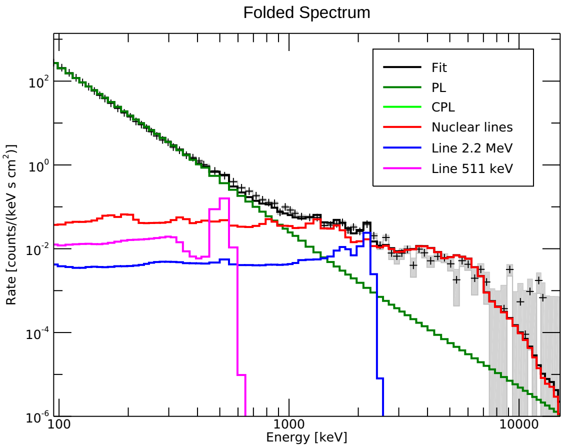




[1] [Lysenko et al., 2019]

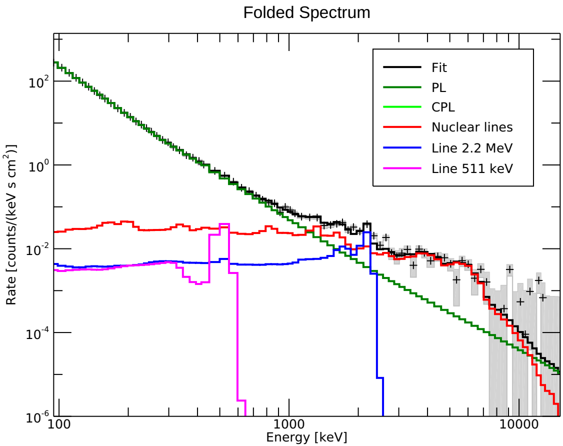
Fitting a continuum component

Power Law



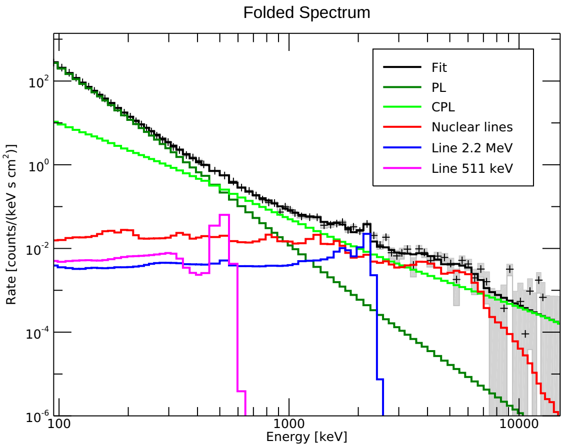
Fitting a continuum component

Broken Power Law



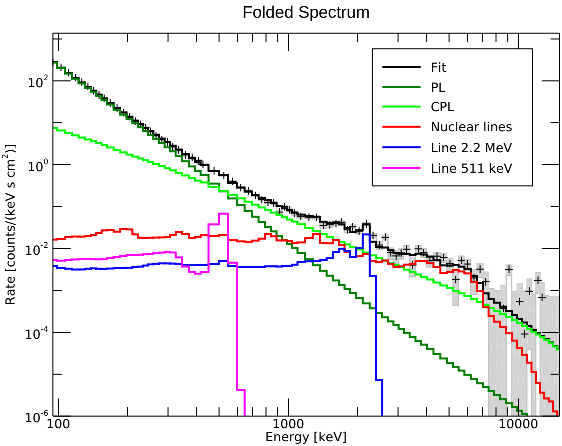
Fitting a continuum component

Sum of two Power Laws



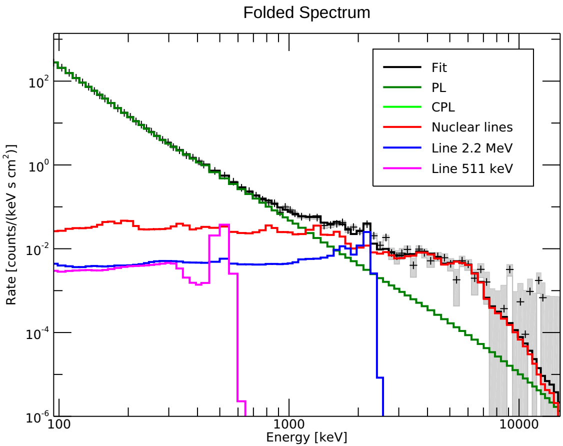
Fitting a continuum component

Power Law + Power Law with cut-off



Fitting a continuum component

Broken Power Law with exponential cut-off



Fitting a continuum component

Bayesian model comparison

No	Model	$\ln Z$	Probability from measurements
1	BPL	-173	$\mathbf{0.76}$
2	BPLexp	-174	$\mathbf{0.24}$
3	PL	-340	$\mathbf{0}$[2]
4	PL + PL2	-181	$\mathbf{0}$[2]
5	PL + CPL	-183	$\mathbf{0}$[2]

[2] below $\mathbf{10^{-3}}$

Fitting a continuum component

Bayesian model comparison

No	Model	$\ln Z$	Likelihood	Prior[2]	Posterior
1	BPL	-173	$\mathbf{0.76}$	$\mathbf{0.05}$	$\mathbf{0.14}$
2	BPLexp	-174	$\mathbf{0.24}$	$\mathbf{0.95}$	$\mathbf{0.86}$
3	PL	-340	$\mathbf{0}$[3]	$\mathbf{0.05}$	$\mathbf{0}$
4	PL + PL2	-181	$\mathbf{0}$[3]	$\mathbf{0.05}$	$\mathbf{0}$
5	PL + CPL	-183	$\mathbf{0}$[3]	$\mathbf{0.95}$	$\mathbf{0}$

[2] Models with exponential cut-off are preferable (e.g. [Ackermann et al., 2012])

[3] below $\mathbf{10^{-3}}$

Fitting a continuum component

Histograms

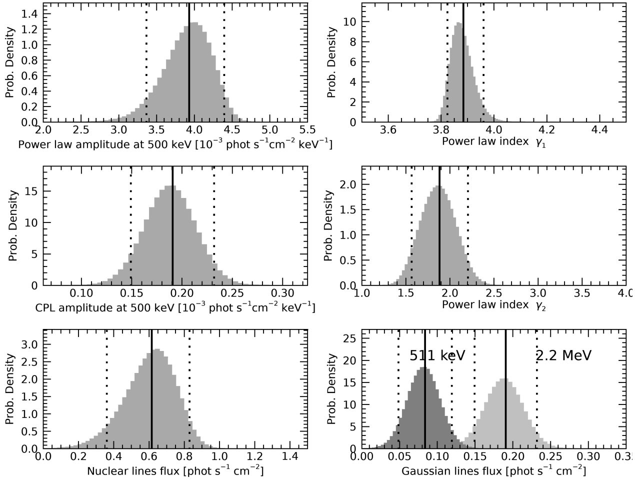


Figure: Histograms for PL+CPL model

Fitting a continuum component

Histograms

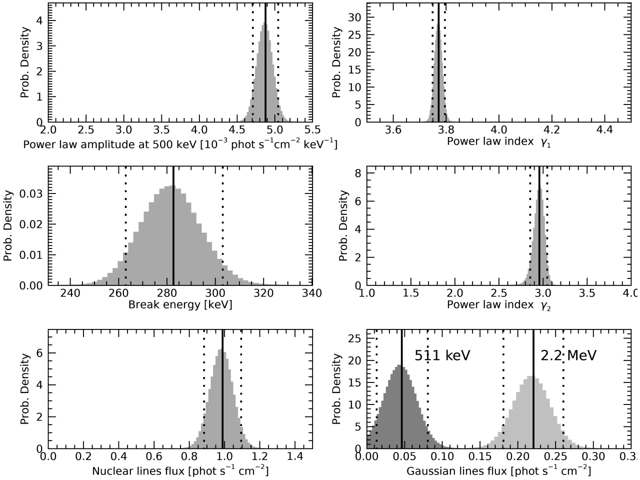
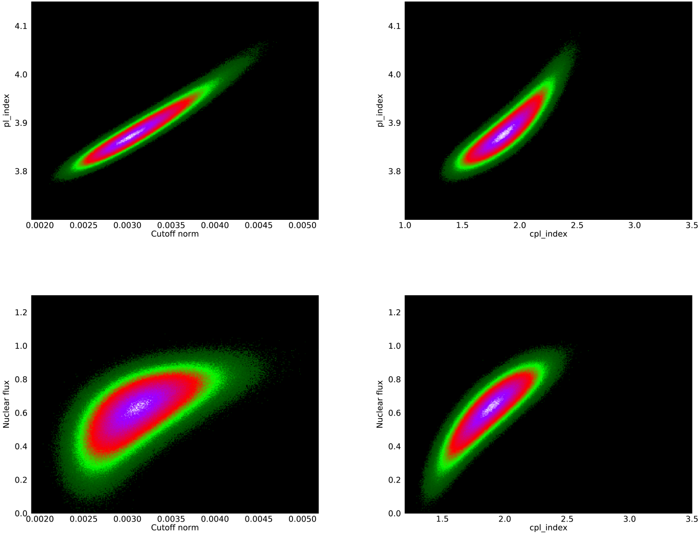


Figure: Histograms for BPOW_EXP model

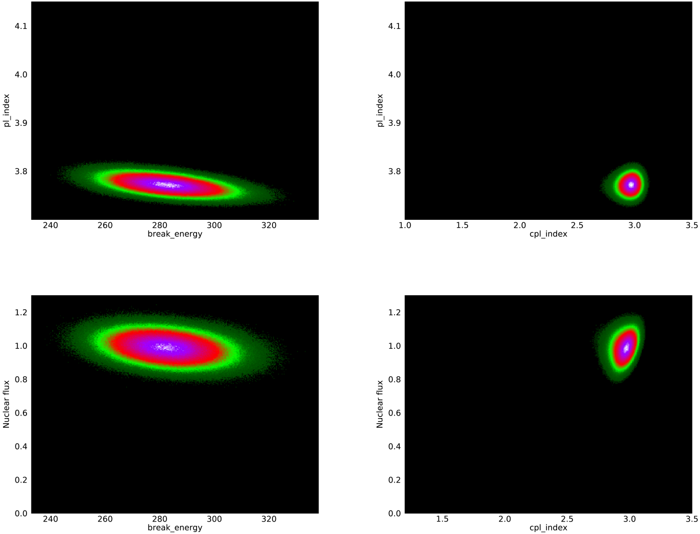
Fitting a continuum component PL+CPL

2D histograms



Fitting a continuum component BPL with cut-off

2D histograms



Presence of components

Bayesian comparison

No	511 keV	2.2 MeV	Nuclear	$\ln Z$	Prob.
1	+	+	+	-173	**0.056**
2	+	+	-	-256	0[4]
3	+	-	+	-208	0[4]
4	+	-	-	-289	0[4]
5	-	+	+	-170	**0.944**
6	-	+	-	-252	0[4]
7	-	-	+	-205	0[4]
8	-	-	-	-320	0[4]

[4] below 10^{-15}

Presence of components

Bayesian comparison

No	511 keV	2.2 MeV	Nucl.	$\ln Z$	Likelihood.	Prior	Post.
1	+	+	+	-173	0.06	0.99	**0.85**
2	+	+	-	-256	0[4]	0.01	0
3	+	-	+	-208	0[4]	0.01	0
4	+	-	-	-289	0[4]	0.01	0
5	-	+	+	-170	0.94	0.01	**0.15**
6	-	+	-	-252	0[4]	0.01	0
7	-	-	+	-205	0[4]	0.01	0
8	-	-	-	-320	0[4]	0.99	0

[4] below $\mathbf{10^{-15}}$

Summary

- Bayesian inference is a universal and robust method for solving inverse problems allowing
  - Inferring model parameters
  - reliable uncertainties estimation
  - quantitative model comparison (comparing rather models than best fits)
- We successfully analysed KW data
  - Superposition of two PLs implies a cross talk between them. Therefore a broken power law model is preferable for describing HXR continuum.
  - Bayesian analysis confirmed presence of accelerated ions in X9.3 flare on 6 September 2017.
  - Details will be given in the talk by Alexandra Lysenko

Many thanks!

- to Alexandra Lysenko, Gregory Fleishman, Dmitry Svinkin and Dmitry Frederiks;
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- to Russian Scientific Foundation who supported this study under grant No 18-72-00144;
- to everyone for listening.

Thank you for your attention!

Take home message:

Bayesian analysis is not a “black magic”. Let us use it to obtain all available information from observations of solar flares and GRB.

Solar Bayesian Analysis Toolkit

Analysis was done with the SoBAT[5] MCMC code written in IDL and allowing for

- MCMC sampling of a user defined PDF
- Sampling Posterior predictive distribution
- Calculating Bayesian evidence for quantitative model comparison
- Easy to use high level routines for fitting $y = f(x) + N(0, \sigma)$ dependencies.
- Predefined and custom priors for free parameters.

[5] Solar Bayesian Analysis Toolkit (SoBAT) available at `https://github.com/Sergey-Anfinogentov/SoBAT`

References

Ackermann, M., Ajello, M., Allafort, A., Atwood, W. B., Baldini, L., Barbiellini, G., Bastieri, D., Bechtol, K., Bellazzini, R., Bhat, P. N., Bland ford, R. D., Bonamente, E., Borgland, A. W., Bregeon, J., Briggs, M. S., Brigida, M., Bruel, P., Buehler, R., Burgess, J. M., Buson, S., Caliand ro, G. A., Cameron, R. A., Casandjian, J. M., Cecchi, C., Charles, E., Chekhtman, A., Chiang, J., Ciprini, S., Claus, R., Cohen-Tanugi, J., Connaughton, V., Conrad, J., Cutini, S., Dennis, B. R., de Palma, F., Dermer, C. D., Digel, S. W., Silva, E. d. C. e., Drell, P. S., Drlica-Wagner, A., Dubois, R., Favuzzi, C., Fegan, S. J., Ferrara, E. C., Fortin, P., Fukazawa, Y., Fusco, P., Gargano, F., Germani, S., Giglietto, N., Giordano, F., Giroletti, M., Glanzman, T., Godfrey, G., Grillo, L., Grove, J. E., Gruber, D., Guiriec, S., Hadasch, D., Hayashida, M., Hays, E., Horan, D., Iafrate, G., Jóhannesson, G., Johnson, A. S., Johnson, W. N., Kamae, T., Kippen, R. M., Knödlseder, J., Kuss, M., Land e, J., Latronico, L., Longo, F., Loparco, F., Lott, B., Lovellette, M. N., Lubrano, P., Mazziotta, M. N., McEnery, J. E., Meegan, C., Mehault, J., Michelson, P. F., Mitthumsiri, W., Monte, C., Monzani, M. E., Morselli, A., Moskalenko, I. V., Murgia, S., Murphy, R., Naumann-Godo, M., Nuss, E., Nymark, T., Ohno, M., Ohsugi, T., Okumura, A., Omodei, N., Orlando, E., Paciesas, W. S., Panetta, J. H., Parent, D., Pesce-Rollins, M., Petrosian, V., Pierbattista, M., Piron, F., Pivato, G.