Anisotropic heat transfer simulation in outer layers of magnetized neutron stars

Kondratyev I.A., Moiseenko S.G., Bisnovatyi-Kogan G.S., Glushikhina M.V.

Space Research Institute RAS

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Observational data

Table 1. Properties of isolated neutron stars observed by ROSAT, Chandra, and XMM-Newton: Pons et al. (2002); Haberl (2004); Haberl et al. (2004); Kaplan et al. (2003); Kaplan & van Kerkwijk (2005).

Source	kT (eV)	P (s)	$\dot{P}$ 10^{-12} (s s^{-1})	τ 10^6 yr	Optical	Optical excess factor	Pulsation amplitude	E_{line} (keV)	B_{db}/B_{cyc} 10^{13} G
RX J0420.0−5022	45	3.453	<9		$B = 26.6$	<12	0.12	0.329	<18/6.6
RX J0720.4−3125	85	8.391	0.07	0.6–2	$B = 26.6$	6	0.11	0.270	2.4/5.2
RX J0806.4−4123	96	11.371	<2		$B > 24$		0.06	–	<14/?
1RXS J130848.6+212708/RBS1223	95	10.313	<6		28.6	<5	0.18	0.3	?/2-6
RX J1605.3+3249	95	–	–		$B = 27.2$	11–14	<0.03	0.46	?/9.5
RX J1856.4−3754	60	–	–	0.5	$V = 25.7$	5–7	<0.02	–	–
1RXS J214303.7+065419/RBS1774	101	9.437	–		$R > 23$		0.04	0.70	?/14

Perez-Azorin, J. F., Miralles, J. A., & Pons, J. A. 2006, A&A, 451, 1009

Yakovlev, D. G. & Pethick, C. J. 2004, ARA&A, 42, 169

- Periodic changes on thermal X-ray light curves of some single neutron stars (NSs) indicate its non-uniform surface temperature distribution
- A possible cause of this phenomenon is a heat conductivity suppression across a magnetic field in outer layers of a magnetized NS
- On the table – observational data from 7 thermally emitting NSs (“magnificent seven”)
- ~~MAGNIFICENT SEVEN~~ --> MAGNIFICENT EIGHT!!! Due to discovery of a thermal component in PSR J0726-2612 Rigoselli et al., 2019

Aim of this study

- An aim of this work is to study three-dimensional effects in a heat transfer in 3D magnetic field configurations (e.g. a non-coaxial superposition of dipolar and quadrupolar fields) and synthesize thermal light curves to find features on them, which correspond a presence of a quadrupolar component

Heat transfer in NSs

- The NS core is considered to be isothermal
- A crustal matter is in a state of a coulomb crystal, pressure and thermal conductivity are determined mostly by strongly degenerate ultrarelativistic electrons
- In the envelope the heat transfer is determined by electrons (and by a radiation in region near the surface)
- We separate outer layers into the crust with densities $\rho = 10^{10} \div 2 \cdot 10^{14}$ and the envelope with $\rho = \rho_s \div 10^{10} g/cm^3$, where $\rho_s \sim 1$ is the density on a NS photosphere
- Crust – 3D heat transfer equation
- Envelope – local plane-parallel model is built
- during a problem solving we make a solution self-consistent

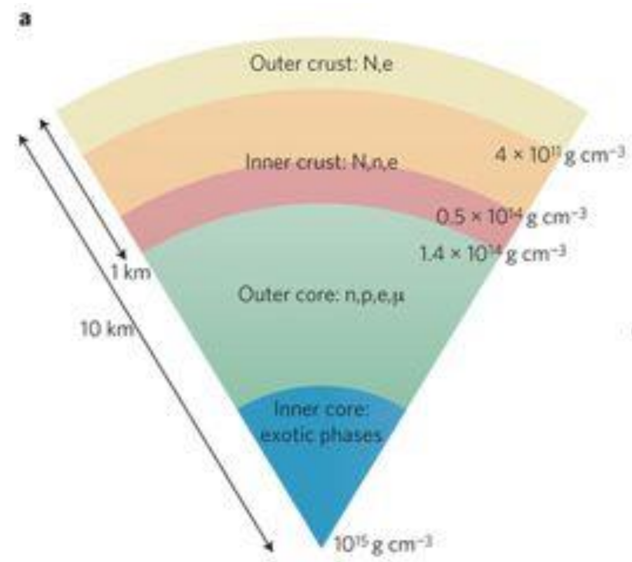


Image: W.G. Newton, Nature physics, 2013

Anisotropic heat transfer in the NS crust

- In the crust a temperature satisfies the heat transfer equation

$$C\frac{\partial T}{\partial t} = \nabla \cdot \hat{\kappa} \cdot \nabla T + f$$

With a **thermal conductivity tensor** (*)

$$\kappa_{ij} = \frac{k_B^2 T n_e}{m_e^*}\tau\Big(\lambda^{(1)}\delta_{ij} + \lambda^{(2)}\varepsilon_{ijk}\frac{B_k}{B} + \lambda^{(3)}\frac{B_i B_j}{B^2}\Big)$$

$$\lambda^{(1)} = \frac{5\pi^2}{6}\Big(\frac{1}{1+(\omega\tau)^2} - \frac{6}{5}\frac{(\omega\tau)^2}{(1+(\omega\tau)^2)^2}\Big)$$

$$\lambda^{(2)} = -\frac{4\pi^2}{3}\omega\tau\Big(\frac{1}{1+(\omega\tau)^2} - \frac{3}{4}\frac{(\omega\tau)^2}{(1+(\omega\tau)^2)^2}\Big)$$

$$\lambda^{(3)} = \frac{5\pi^2}{6}(\omega\tau)^2\Big(\frac{1}{1+(\omega\tau)^2} - \frac{6}{5}\frac{1}{(1+(\omega\tau)^2)^2}\Big)$$

Which was derived as a Boltzmann equation solution with a Chapman-Enskog method in a Lorentz approximation. This tensor has a much more complicated dependence on the magnetic field than in previous works, where the heat flux suppression is taken into account phenomenologically.

(*) *G. S. Bisnovatyi-Kogan, M. V. Glushikhina* Plasma Physics Reports, 2018, Vol. 44, No. 4, pp. 405-423

Thermal conductivity differences

- Previous studies (average velocity is neglected (!), diffusion is taken into account)

$$\kappa'_{e\|} = \frac{\pi^2}{3}\frac{k_B^2 T n_e}{m_e^*}\tau$$

- Our study (average velocity is not equal to zero, diffusion is neglected)

$$\kappa_{e\|} = \frac{5\pi^2}{6}\frac{k_B^2 T n_e}{m_e^*}\tau$$

Thermal structure equation for the NS envelope

- In a first approach, radial heat flux of the envelope is much stronger, than the flux along the envelope, $F_n \gg F_\tau$. Under this assumption a thermal structure equation can be written in a **local, one-dimensional, plane-parallel approximation**
- Heat transfer and hydrostatic equilibrium equations lead to

$$\frac{dT}{dP} = \frac{3K}{16g}\frac{T_s^4}{T^3}$$

Here ***P*** – pressure, ***g*** – NS surface gravity acceleration, ***K*** – opacity, $\boldsymbol{T_s}$ - local surface temperature

Integration of this equation from the surface to the bottom of the envelope with $\rho_b = 10^{10} g/cm^3$ gives the local temperature T_b on the crust-envelope interface

Effective opacity

$$\boldsymbol{K^{-1} = K_e^{-1} + K_r^{-1};} \quad \boldsymbol{\kappa = \kappa_e + \kappa_r}$$

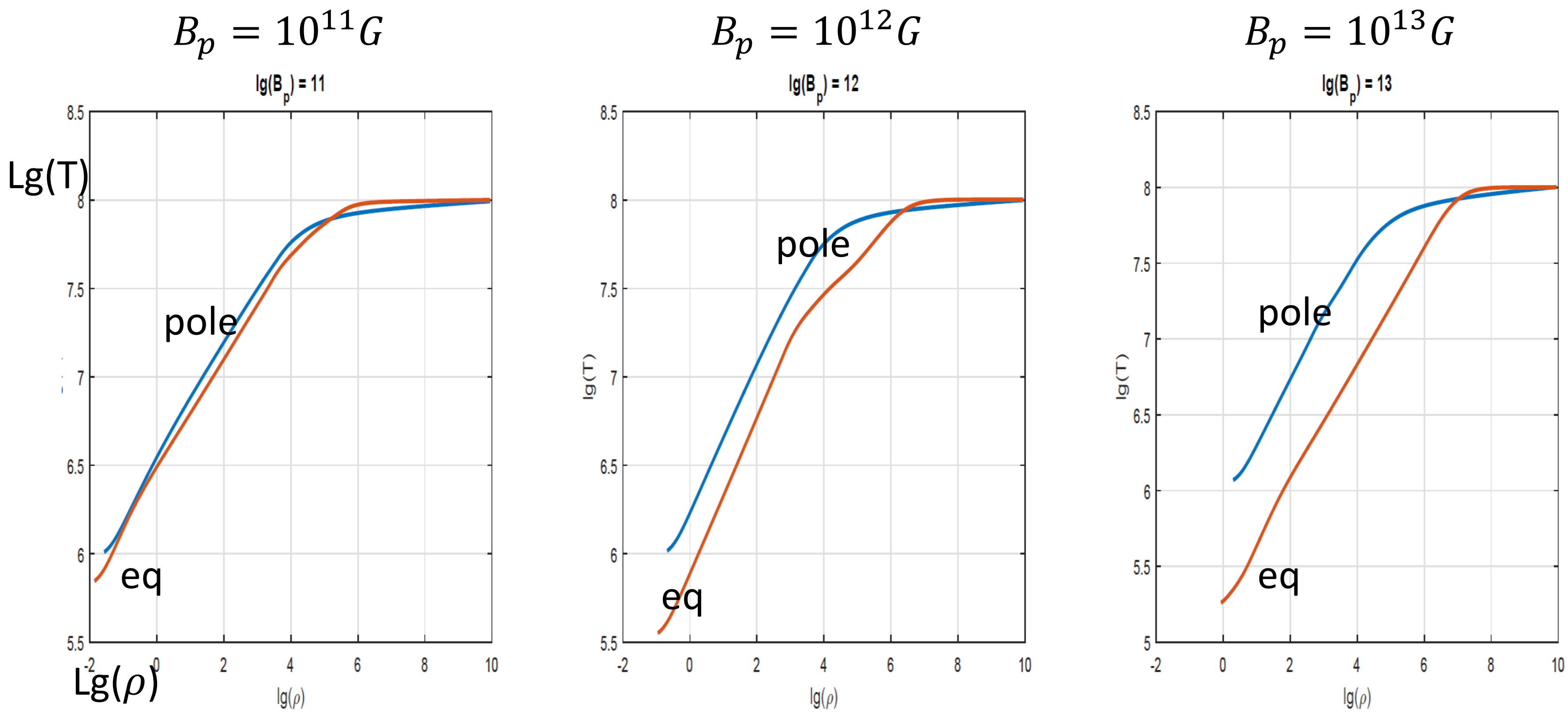
$\boldsymbol{\kappa = \kappa_{\parallel}\cos^2\theta_B + \kappa_{\perp}\sin^2\theta_B}$ - thermal conductivity coefficient (electron and radiative) in the magnetic field, θ_B is a field inclination angle.

- Corresponding opacities are as follows:

Electron: $K_e = \frac{16\sigma T^3}{3\kappa_e \rho}$

Radiative: $K_r = K_{ff} + K_{bf} + K_{Th}$

Thermal structure of the magnetized envelope



$\lg B_p$	11	12	13
$T_s(\theta = 0)/10^6 K$	1.02	1.03	1.16
$T_s(\theta = \pi/2)/10^6 K$	0.71	0.35	0.18
$T_{s\|}/T_{s\perp}$	1.43	2.94	6.44

$\boldsymbol{T_b = 10^8 K}$

By varying T_s, θ and B, the **“$T_s - T_b$” relationship** can be obtained. It is a dependendce of surface temperature on the crust-envelope one, $\boldsymbol{T_s = T_s(T_b)}$.

A boundary-value problem for the heat transfer equation in the NS crust

- We look for a stationary solution due to the slow cooling
- The heat flux in the envelope is assumed to be radial
- Isothermal NS core with the temperature T_{core}

The boundary value problem:

$$\nabla \cdot \kappa(\mathrm{B}, \rho, T) \cdot \nabla T = 0$$

$$T\big|_{in} = T_{core}, \quad \kappa(\mathrm{B}, \rho, T) \nabla_r T + F_s\big|_{out} = 0$$

Additionally

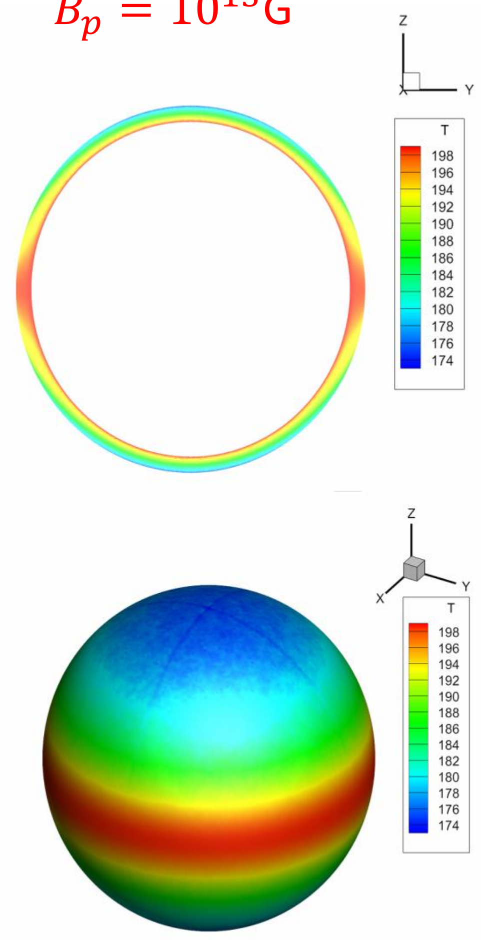
$$F_s = \sigma(T_s(\theta_B, \mathrm{B}, T_b))^4$$

$$\cos(\theta_B) = \frac{\boldsymbol{B} \cdot \boldsymbol{r}}{Br}$$

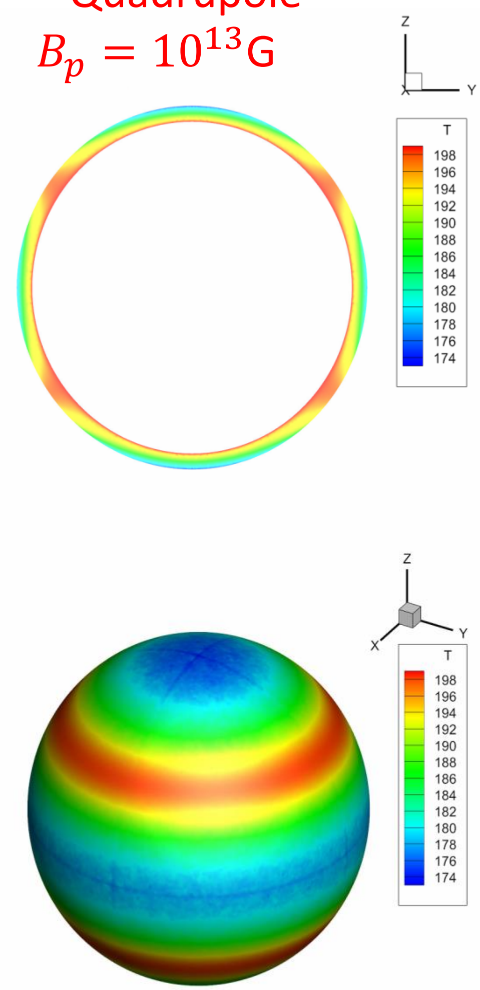
Computation results (crust)

- The NS core temperature $T_{core} = 2 \cdot 10^8 K$

Dipole
$B_p = 10^{13}$G

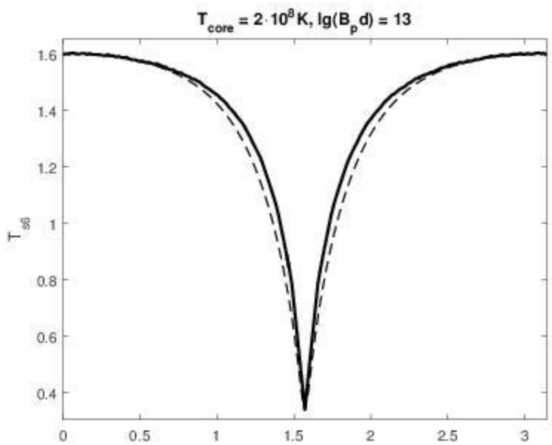


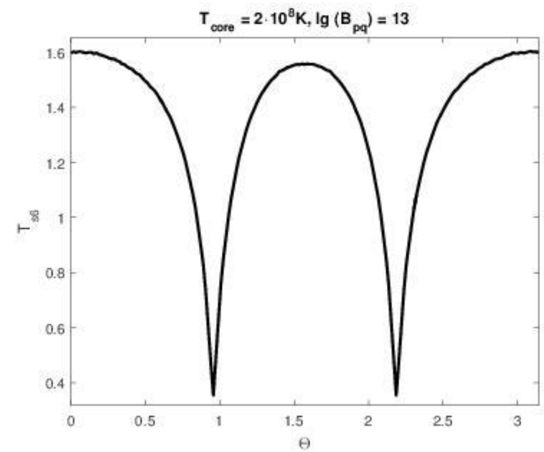
Quadrupole
$B_p = 10^{13}$G

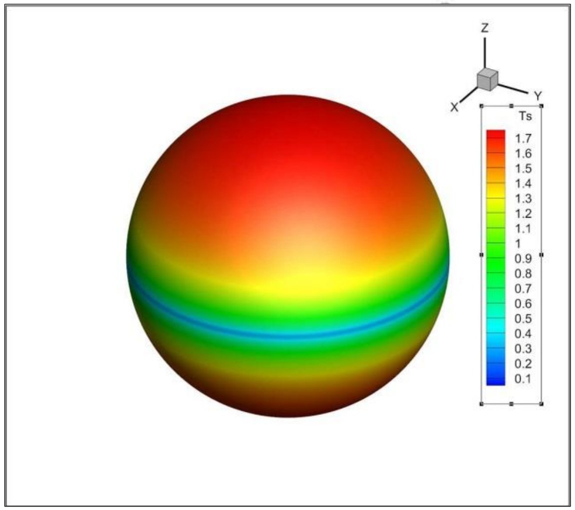


Computation results (surface)

- Surface temperature distribution







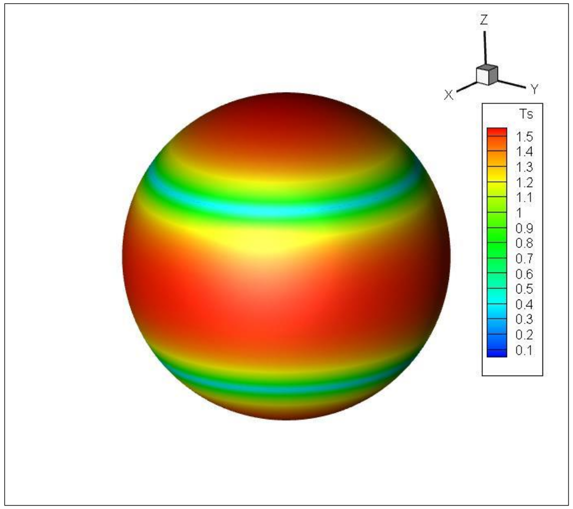
Dipole

$B_p = 10^{13} G$

T-distribution =
Hot polar caps +
cold belt(s)

$T_{s(max)} \approx 1.6 \cdot 10^6 K$

$T_{s(min)} \approx 0.3 \cdot 10^6 K$



Quadrupole

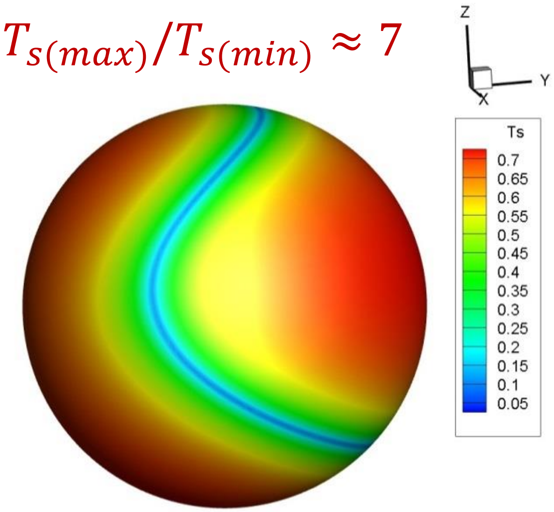
$B_p = 10^{13} G$

Computation results (+quadrupole)

- Now we "switch on" a quadrupolar field!

- Physical parameters: $\Theta_b = \angle(\boldsymbol{B}_{dip}, \boldsymbol{B}_{quad});\quad \beta = \frac{B_{quad}}{B_{dip}};$

$T_{s(max)}/T_{s(min)} \approx 7$

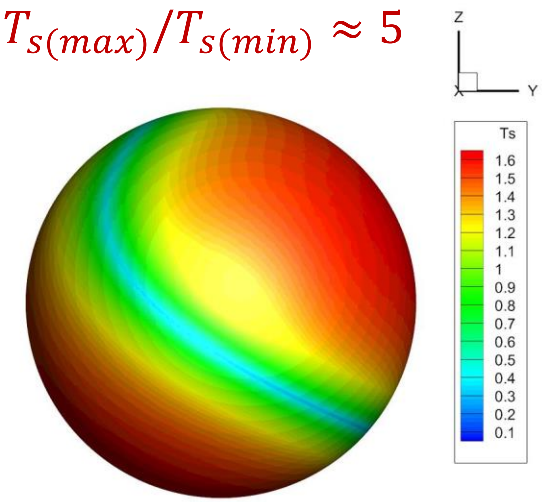


$T_{core} = 5 \cdot 10^7 K$

$\Theta_b = 60^\circ$

$\beta = 0.75$

$T_{s(max)}/T_{s(min)} \approx 5$

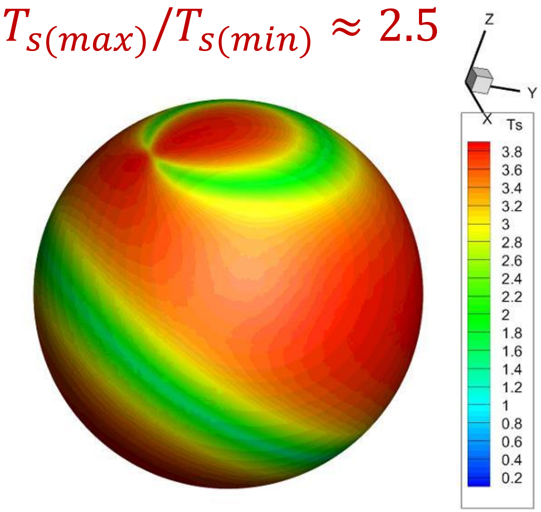


$T_{core} = 2 \cdot 10^8 K$

$\Theta_b = 45^\circ$

$\beta = 0.5$

$T_{s(max)}/T_{s(min)} \approx 2.5$



$T_{core} = 1 \cdot 10^9 K$

$\Theta_b = 30^\circ$

$\beta = 1$

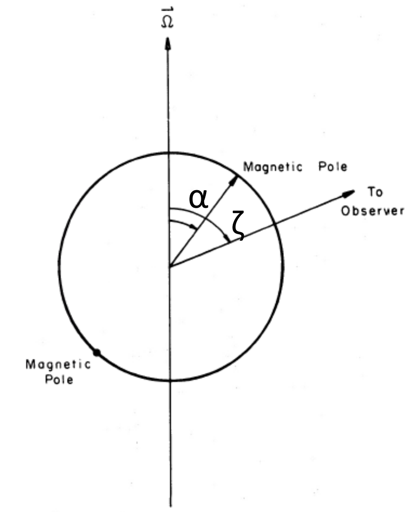
Main results for the NS surface temperature

- For $\boldsymbol{\beta} \gg \mathbf{1}$ and $\boldsymbol{\beta} \ll \mathbf{1}$ the temperature distribution approaches to the pure-quadrupolar and dipolar ones correspondingly
- The presence of the non-coaxial quadrupolar field makes the cold belt take an irregular shape (“jaw”)
- At moderate parameters ($\boldsymbol{\Theta_b} \sim \mathbf{45}^{\circ}, \boldsymbol{\beta} \sim \mathbf{0.5}$) the belt broadens, and the cold region becomes larger
- For the strong quadrupolar fields the second belt appears at $\boldsymbol{\beta} \sim \mathbf{1}$ at small angles $\boldsymbol{\Theta_b} \ll \mathbf{90}^{\circ}$ and at $\boldsymbol{\beta} \gtrsim \mathbf{1.5}$ for $\boldsymbol{\Theta_b} \lesssim \mathbf{90}^{\circ}$

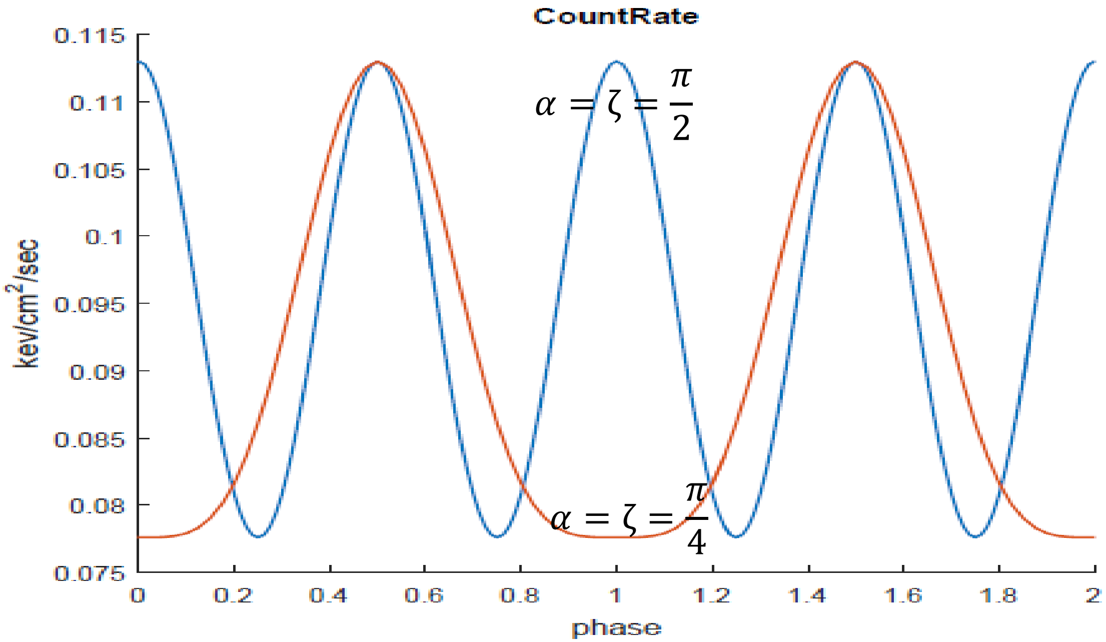
Synthetic light curves

- Composite black-body model (locally equilibrium)
- General relativity effects, taken into account:
    1. **Gravitational redshift**
    2. **Light bending** =>

D.Page 1995, ApJ, 442, 273 an effective NS radius and a body angle increase for an observer at the infinite distance



Light curve (dipolar)



(*) Greenstein G., Hartke G.J. 1983, ApJ, 271, 283

No GR (*):

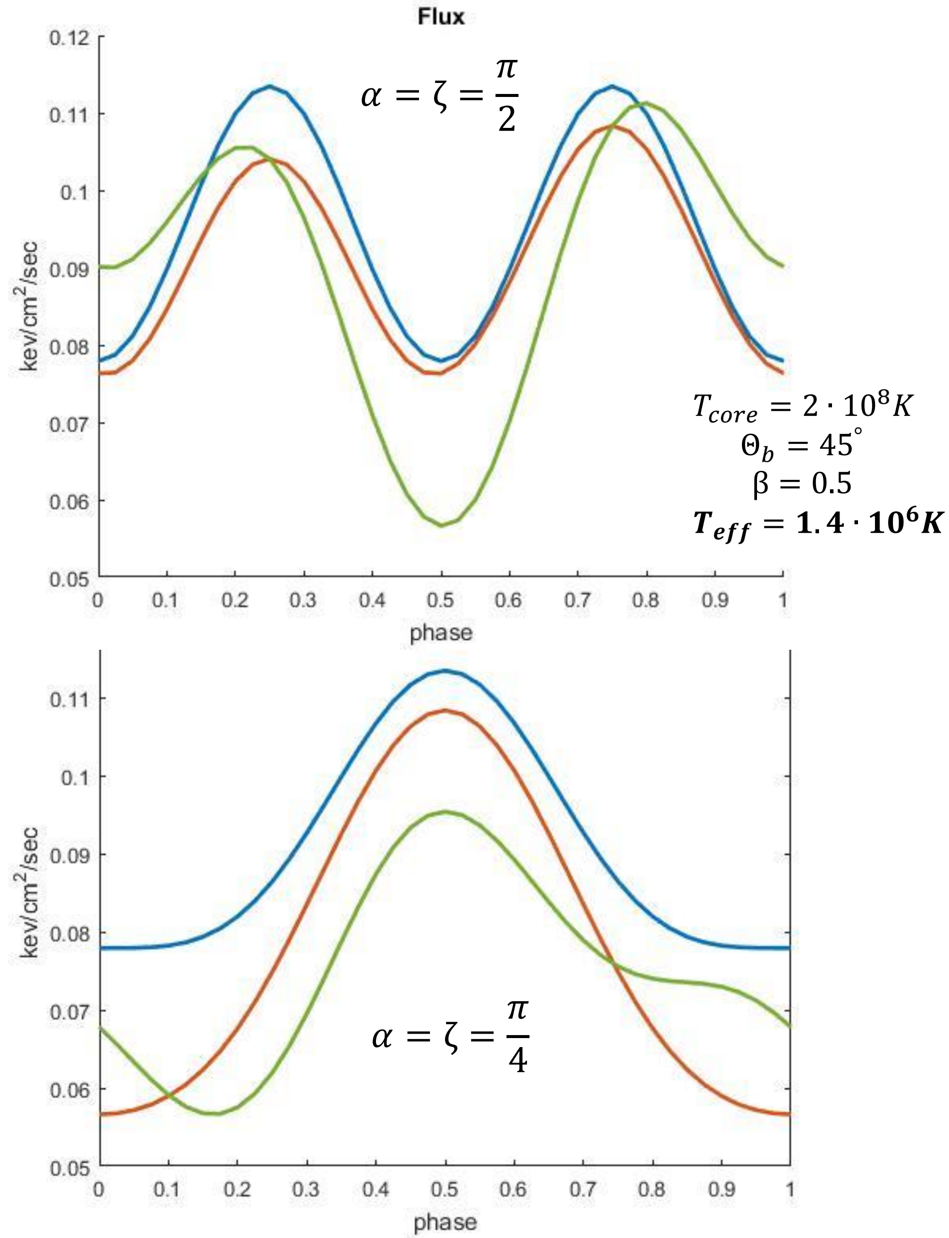
$\alpha + \zeta \leq \frac{\pi}{2}$ – 1peaked

Else – 2peaked

Light curves

Observational manifestations (preliminary)

- Two limits for positions of **rotational**, **quadrupolar** and **dipolar** axes
- Pulsations can be increased sufficiently!
- Peak symmetry is broken, if all three axes are not in the same plane
- Pulse shape may change due to GR effects (2p->1p)



Summary

- We have studied 3D effects in the heat transfer in the magnetized neutron stars
- Surface temperature distribution with inclusion of a quadrupolar field changes sufficiently from the pure-dipolar one
- An existence of the magnetic fields with no axial (cylindrical) symmetry, in principle, can be detectable due to special features on the thermal light curves

Thank you for attention! ☺

More detailed discussion about The thermal conductivity (backup slide)

- A case along the MF (electron motion is treated as free)
- Boltzmann equation solution is as follows:

$$\mathbf{q}_{\parallel} = -\frac{640 k_B}{\Lambda}\frac{m_e (k_B T)^4}{n_N Z^2 e^4 h^3}\left(G_5 - \frac{1}{2}\frac{G_{5/2}}{G_{3/2}} G_4\right)\cdot \nabla T - \frac{128}{\Lambda}\frac{m_e (k_B T)^5}{n_N Z^2 e^4 h^3}\frac{G_{5/2}}{G_{3/2}} G_4 \cdot \mathbf{d_e}$$

$$\langle \mathbf{v_e} \rangle = -\frac{128 k_B}{\Lambda}\frac{m_e (k_B T)^3}{n_N n_e Z^2 e^4 h^3}\left(G_4 - \frac{5}{8}\frac{G_{5/2}}{G_{3/2}} G_3\right)\cdot \nabla T - \frac{32}{\Lambda}\frac{m_e (k_B T)^4}{n_N n_e Z^2 e^4 h^3}\frac{G_{5/2}}{G_{3/2}} G_3 \cdot \mathbf{d_e},$$

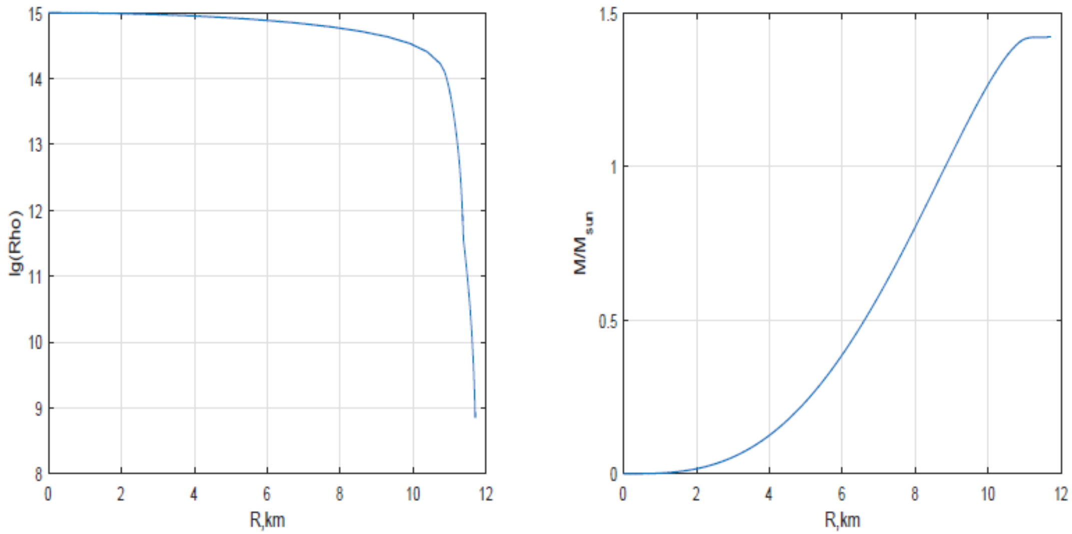
- Assuming V_e = 0 leads to pi^2/3
- Assuming d_e = 0 leads to 5pi^2/6

Neglecting <u>diffusion</u> in the NS interior is approximate, but neglecting <u>electric currents</u> is valid only in terrestrial conditions (e.g. metals)

NS model and equation of state (backup slide)

- In the core and the crust of the NS a **unified equation of state SLy4** (moderately stiff) is used. It is based on calculations with an effective nuclear potential with zero temperature. It was calculated consistently for the core and the crust. **A core composition is neutrons, protons, electrons and muons**.

Tolman-Oppenheimer-Volkoff equations solutions



$$M_{NS} = 1.42 M_{sun}$$
$$R_{NS} = 11.62 km$$
$$\rho_c = 1 \cdot 10^{15} g/cm^3$$

- The matter in the NS envelope is treated as an **ideal completely ionized plasma** of degenerate relativistic electrons (pressure is represented by analytical approximations of Fermi-Dirac integrals for arbitrary degrees of degeneracy and relativism) and non-degenerate non-relativistic ions, **effect of magnetic field on EOS is not taken into account**

Numerical technique (backup slide)

- Operator approach in finite-difference schemes (Basic operators method)

Kondratyev I.A., Moiseenko S.G. 2019,JPCS,1163 012069

- GRAD operator (nodes -> cells):

$$(\nabla_{\triangle} p)_i = \frac{1}{V_i} \sum_{k=1}^{4} (\bar{p}_k S_k \vec{n}_k)_i$$

- Green Formula and its analogue:

$$\int p \nabla \cdot \vec{v} dV + \int \vec{v} \cdot \nabla p dV = 0$$

$$\sum_{l=1}^{N_l} \nabla_{\times} \cdot \vec{v}_l p_l W_l = - \sum_{k=1}^{K_j} \vec{v}_k \nabla_{\triangle} p_k V_k$$

- DIV operator (cells -> nodes):

$$(\nabla_{\times} \cdot \vec{v})_j = - \frac{1}{3W_j} \sum_{k=1}^{K_j} \tilde{\vec{v}}_k \cdot (\vec{n}_1 S_1 + \vec{n}_2 S_2 + \vec{n}_3 S_3)_k$$